

## II. ДИФФЕРЕНЦИРОВАНИЕ

### Теоретические вопросы

1. Понятие производной. Производная функции  $x^n$ .
2. Геометрический смысл производной. Уравнения касательной и нормали к графику функции.
3. Понятие дифференцируемости функции и дифференциала. Условие дифференцируемости. Связь дифференциала с производной.
4. Геометрический смысл дифференциала.
5. Непрерывность дифференцируемой функции.
6. Дифференцирование постоянной и суммы, произведения и частного.
7. Производная сложной функции.
8. Инвариантность формы дифференциала.
9. Производная обратной функции.
10. Производные обратных тригонометрических функций.
11. Гиперболические функции, их производные.
12. Производные высших порядков, формула Лейбница.
13. Дифференциалы высших порядков. Неинвариантность дифференциалов порядка выше первого.
14. Дифференцирование функций, заданных параметрически.

### Теоретические упражнения

1. Исходя из определения производной, доказать, что
  - а) производная периодической дифференцируемой функции есть функция периодическая;
  - б) производная четной дифференцируемой функции есть функция нечетная;
  - в) производная нечетной дифференцируемой функции есть функция четная.
2. Доказать, что если функция  $f(x)$  дифференцируема в точке  $x = 0$  и  $f(0) = 0$ , то
$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x)}{x}.$$
3. Доказать, что производная  $f'(0)$  не существует, если

$$4. f(x) = \begin{cases} x \sin(1/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

5. Доказать, что производная от функции

$$6. f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

7. разрывна в точке  $x = 0$ .

8. Доказать приближенную формулу

$$a. \sqrt{a^2 + z} \approx a + z/(2a), \quad a > 0, \quad |z| \ll a.$$

9. Что можно сказать о дифференцируемости суммы  $f(x) + g(x)$  в точке  $x = x_0$  если, в этой точке:

10. а) функция  $f(x)$  дифференцируема, а функция  $g(x)$  не дифференцируема;

11. б) обе функции  $f(x)$  и  $g(x)$  не дифференцируемы.

12. Пусть функция  $f(x)$  дифференцируема в точке  $x_0$  и  $f'(x_0) \neq 0$ , а функция  $g(x)$  не дифференцируема в этой точке. Доказать, что произведение  $f(x)g(x)$  является недифференцируемым в точке  $x_0$ .

13. Что можно сказать о дифференцируемости произведения  $f(x)g(x)$  в предположениях задачи?

а. Рассмотреть примеры:

б. а)  $f(x) = x, \quad g(x) = |x|, \quad x_0 = 0;$

в.  $f(x) = x, \quad g(x) = \begin{cases} \sin(1/x), & x \neq 0, \\ 0, & x = 0, \end{cases} \quad x_0 = 0;$

д. б)  $f(x) = |x|, \quad g(x) = |x|, \quad x_0 = 0;$

е.  $f(x) = |x|, \quad g(x) = |x| + 1, \quad x_0 = 0.$

14. Найти  $f'(0)$ , если  $f(x) = x(x+1)\dots(x+1234567)$ .

15. Выразить дифференциал  $d^3y$  от сложной функции  $y[u(x)]$  через производные от функции  $y(u)$  и дифференциалы от функции  $u(x)$ .

16. Пусть  $y(x)$  и  $x(y)$  дважды дифференцируемые взаимно обратные функции. Выразить  $x''$  через  $y'$  и  $y''$ .

### Расчетные задания

**Задача 1.** Исходя из определения производной, найти  $f'(0)$ .

$$1.1. f(x) = \begin{cases} \operatorname{tg}\left(x^3 + x^2 \sin \frac{2}{x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.2. f(x) = \begin{cases} \arcsin\left(x^2 \cos \frac{1}{9x}\right) + \frac{2}{3}x, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.3. f(x) = \begin{cases} \operatorname{arctg}\left(x \cos \frac{1}{5x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.4. f(x) = \begin{cases} \ln\left(1 - \sin\left(x^3 \sin \frac{1}{x}\right)\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.5. f(x) = \begin{cases} \sin\left(x \sin \frac{3}{x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.6. f(x) = \begin{cases} \sqrt{1 + \ln\left(1 + x^2 \sin \frac{1}{x}\right)} - 1, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.7. f(x) = \begin{cases} \sin\left(e^{x^2 \sin \frac{5}{x}} - 1\right) + x, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.8. f(x) = \begin{cases} x^2 \cos \frac{4}{3x} + \frac{x^2}{2}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.9. f(x) = \begin{cases} \operatorname{arctg}\left(x^3 - x^{\frac{3}{2}} \sin \frac{1}{3x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.10. f(x) = \begin{cases} \sin x \cdot \cos \frac{5}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.11. f(x) = \begin{cases} x + \arcsin\left(x^2 \sin \frac{6}{x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.12. f(x) = \begin{cases} \operatorname{tg}\left(2^{x^2 \cos(1/8x)} - 1 + x\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.13. f(x) = \begin{cases} \operatorname{arctg} x \cdot \sin \frac{7}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.14. f(x) = \begin{cases} 2x^2 + x^2 \cos \frac{1}{9x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.15. f(x) = \begin{cases} x^2 \cos^2 \frac{11}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.16. f(x) = \begin{cases} 2x^2 + x^2 \cos \frac{1}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.17. f(x) = \begin{cases} \frac{\ln \cos x}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.18. f(x) = \begin{cases} 6x + x \sin \frac{1}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.19. f(x) = \begin{cases} \frac{e^{x^2} - \cos x}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.20. f(x) = \begin{cases} e^{x \sin \frac{5}{x}} - 1, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.21. f(x) = \begin{cases} 3^{x^2 \sin \frac{2}{x}} - 1 + 2x, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.22. f(x) = \begin{cases} \sqrt{1 + \ln \left( 1 + 3x^2 \cos \frac{2}{x} \right)} - 1, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.23. f(x) = \begin{cases} e^{x \sin \frac{3}{5x}} - 1, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.24. f(x) = \begin{cases} \frac{2 \operatorname{tg} x - 2 \sin x}{x^2}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.25. f(x) = \begin{cases} \operatorname{arctg} \left( \frac{3x}{2} - x^2 \sin \frac{1}{x} \right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.26. f(x) = \begin{cases} e^{\sin \left( x^{\frac{3}{2}} \sin \frac{2}{x} \right)} - 1 + x^2, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.27. f(x) = \begin{cases} \sqrt[3]{1 - 2x^3 \sin \frac{5}{x}} - 1 + x, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.28. f(x) = \begin{cases} x^2 e^{|x|} \sin \frac{1}{x^2}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.29. f(x) = \begin{cases} \frac{\ln(1 + 2x^2 + x^3)}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

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$$1.30. f(x) = \begin{cases} \frac{\cos x - \cos 3x}{x}, & x \neq 0; \\ 0, & x = 0. \end{cases}$$

$$1.31. f(x) = \begin{cases} 1 - \cos\left(x \sin \frac{1}{x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

**Задача 2.** Составить уравнение нормали (в вариантах 2.1 – 2.12) или уравнение касательной (в вариантах 2.13 – 2.31) к данной кривой в точке с абсциссой  $x_0$ .

$$2.1. y = (4x - x^2)/4, \quad x_0 = 2.$$

$$2.2. y = 2x^2 + 3x - 1, \quad x_0 = -2.$$

$$2.3. y = x - x^3, \quad x_0 = -1.$$

$$2.4. y = x^2 + 8\sqrt{x} - 32, \quad x_0 = 4.$$

$$2.5. y = x + \sqrt{x^3}, \quad x_0 = 1.$$

$$2.6. y = \sqrt[3]{x^2} - 20, \quad x_0 = -8.$$

$$2.7. y = \frac{1 + \sqrt{x}}{1 - \sqrt{x}}, \quad x_0 = 4.$$

$$2.8. y = 8\sqrt[4]{x} - 70, \quad x_0 = 16.$$

$$2.9. y = 2x^2 - 3x + 1, \quad x_0 = 1.$$

$$2.10. y = (x^2 - 3x + 6)/x^2, \quad x_0 = 3.$$

$$2.11. y = \sqrt{x} - 3\sqrt[3]{x}, \quad x_0 = 64.$$

$$2.12. y = (x^3 + 2)/(x^3 - 2), \quad x_0 = 2.$$

$$2.13. y = 2x^2 + 3, \quad x_0 = 1.$$

$$2.14. y = \frac{x^{29} + 6}{x^4 + 1}, \quad x_0 = 1.$$

$$2.15. y = 2x + \frac{1}{x}, \quad x_0 = 1.$$

$$2.16. y = -2(x^8 + 2)/(3(x^4 + 1)), \quad x_0 = 1.$$

$$2.17. y = \frac{x^5 + 1}{x^4 + 1}, \quad x_0 = 1.$$

$$2.18. y = \frac{x^{16} + 9}{1 - 5x^2}, \quad x_0 = 1.$$

$$2.19. y = 3(\sqrt[3]{x} - 2\sqrt{x}), \quad x_0 = 1.$$

$$2.20. y = 1/(3x + 2), \quad x_0 = 2.$$

$$2.21. y = x/(x^2 + 1), \quad x_0 = -2.$$

$$2.22. y = (x^2 - 3x + 3)/3, \quad x_0 = 3.$$

$$2.23. y = 2x/(x^2 + 1), \quad x_0 = 1.$$

$$2.24. y = -2(\sqrt[3]{x} + 3\sqrt{x}), \quad x_0 = 1.$$

$$2.25. y = \frac{1+3x^2}{3+x^2}, \quad x_0 = 1.$$

$$2.27. y = 3\sqrt[4]{x} - \sqrt{x}, \quad x_0 = 1.$$

$$2.29. y = x^2/10 + 3, \quad x_0 = 2.$$

$$2.31. y = 6\sqrt[3]{x} - 16\sqrt[4]{x}/3, \quad x_0 = 1.$$

$$2.26. y = 14\sqrt{x} - 15\sqrt[3]{x} + 2, \quad x_0 = 1.$$

$$2.28. y = (3x - 2x^3)/3, \quad x_0 = 1.$$

$$2.30. y = (x^2 - 2x - 3)/4, \quad x_0 = 4.$$

**Задача 3.** Найти дифференциал  $dy$ .

$$3.1. y = x \arcsin(1/x) + \ln|x + \sqrt{x^2 - 1}|, \quad x > 0.$$

$$3.3. y = \sqrt{1+2x} - \ln|x + \sqrt{1+2x}|.$$

$$3.5. y = \arccos(1/\sqrt{1+2x^2}), \quad x > 0.$$

$$3.7. y = \operatorname{arctg}(\operatorname{sh} x) + (\operatorname{sh} x) \operatorname{lnch} x.$$

$$3.9. y = \ln(\cos^2 x + \sqrt{1 + \cos^4 x}).$$

$$3.11. y = \frac{\ln|x|}{1+x^2} - \frac{1}{2} \ln \frac{x^2}{1+x^2}$$

$$3.13. y = x\sqrt{4-x^2} + a \arcsin(x/2).$$

$$3.15. y = 2x + \ln|\sin x + 2 \cos x|.$$

$$3.17. y = \ln \left| \frac{x + \sqrt{x^2 + 1}}{2x} \right|.$$

$$3.19. y = \operatorname{arctg} \frac{x^2 - 1}{x}.$$

$$3.2. y = \operatorname{tg}(2 \arccos \sqrt{1-2x^2}), \quad x > 0.$$

$$3.4. y = x^2 \operatorname{arctg} \sqrt{x^2 - 1} - \sqrt{x^2 - 1}.$$

$$3.6. y = x \ln|x + \sqrt{x^2 + 3}| - \sqrt{x^2 + 3}.$$

$$3.8. y = \arccos\left(\frac{(x^2 - 1)}{(x^2 \sqrt{2})}\right).$$

$$3.10. y = \ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2} \operatorname{arctg} x.$$

$$3.12. y = \ln(e^x + \sqrt{e^{2x} - 1}) + \operatorname{arcsine}^x.$$

$$3.14. y = \operatorname{Intg}(x/2) - x/\sin x.$$

$$3.16. y = \sqrt{\operatorname{ctg} x} - \sqrt{\operatorname{tg}^3 x}/3.$$

$$3.18. y = \sqrt[3]{\frac{x+2}{x-2}}.$$

$$3.20. y = \ln|x^2 - 1| - \frac{1}{x^2 - 1}.$$

3.21.  $y = \operatorname{arctg}\left(\operatorname{tg}\frac{x}{2} + 1\right).$

3.22.  $y = \ln\left|2x + 2\sqrt{x^2 + x + 1}\right|.$

3.23.  $y = \ln|\cos\sqrt{x}| + \sqrt{x} \operatorname{tg}\sqrt{x}.$

3.24.  $y = e^x (\cos 2x + 2 \sin 2x).$

3.25.  $y = x(\sin \ln x - \cos \ln x).$

3.26.  $y = \left(\sqrt{x-1} - \frac{1}{2}\right)e^{2\sqrt{x-1}}.$

3.27.  $y = \cos x \cdot \operatorname{Intg} x - \operatorname{Intg}\frac{x}{2}.$

3.28.  $y = \sqrt{3+x^2} - x \ln|x + \sqrt{3+x^2}|.$

3.29.  $y = \sqrt{x} - (1+x) \operatorname{arctg}\sqrt{x}.$

3.30.  $y = x \operatorname{arctg} x - \ln\sqrt{1+x^2}.$

3.31.  $y = x\sqrt{x^2-1} + \ln|x + \sqrt{x^2-1}|.$

**Задача 4.** Вычислить приближенно с помощью дифференциала.

4.1.  $y = \sqrt[3]{x}, \quad x = 7,76.$

4.2.  $y = \sqrt[3]{x^3 + 7x}, \quad x = 1,012.$

4.3.  $y = \left(x + \sqrt{5-x^2}\right)/2, \quad x = 0,98.$

4.4.  $y = \sqrt[3]{x}, \quad x = 27,54.$

4.5.  $y = \arcsin x, \quad x = 0,08.$

4.6.  $y = \sqrt[3]{x^2 + 2x + 5}, \quad x = 0,97.$

4.7.  $y = \sqrt[3]{x}, \quad x = 26,46.$

4.8.  $y = \sqrt{x^2 + x + 3}, \quad x = 1,97.$

4.9.  $y = x^{11}, \quad x = 1,021.$

4.10.  $y = \sqrt[3]{x}, \quad x = 1,21.$

4.11.  $y = x^{21}, \quad x = 0,998.$

4.12.  $y = \sqrt[3]{x^2}, \quad x = 1,03.$

4.13.  $y = x^6, \quad x = 2,01.$

4.14.  $y = \sqrt[3]{x}, \quad x = 8,24.$

4.15.  $y = x^7, \quad x = 1,996.$

4.16.  $y = \sqrt[3]{x}, \quad x = 7,64.$

4.17.  $y = \sqrt{4x-1}, \quad x = 2,56.$

4.18.  $y = 1/\sqrt{2x^2 + x + 1}, \quad x = 1,016.$

4.19.  $y = \sqrt[3]{x}, \quad x = 8,36.$

4.20.  $y = 1/\sqrt{x}, \quad x = 4,16.$

4.21.  $y = x^7, \quad x = 2,002.$

4.22.  $y = \sqrt{4x-3}, \quad x = 1,78.$

4.23.  $y = \sqrt{x^3}, \quad x = 0,98.$

4.24.  $y = x^5, \quad x = 2,997.$



$$4.25. y = \sqrt[5]{x^2}, \quad x = 1,03.$$

$$4.27. y = \sqrt{1+x+\sin x}, \quad x = 0,01.$$

$$4.29. y = \sqrt[4]{2x - \sin(\pi x/2)}, \quad x = 1,02.$$

$$4.31. y = 1/\sqrt{2x+1}, \quad x = 1,58.$$

$$4.26. y = x^4, \quad x = 3,998.$$

$$4.28. y = \sqrt[3]{3x + \cos x}, \quad x = 0,01.$$

$$4.30. y = \sqrt{x^2 + 5}, \quad x = 1,97.$$

**Задача 5.** Найти производную.

$$5.1. y = \frac{2(3x^3 + 4x^2 - x - 2)}{15\sqrt{1+x}}.$$

$$5.3. y = \frac{x^4 - 8x^2}{2(x^2 - 4)}.$$

$$5.5. y = \frac{(1+x^8)\sqrt{1+x^8}}{12x^{12}}.$$

$$5.7. y = \frac{(x^2 - 6)\sqrt{(4+x^2)^3}}{120x^5}.$$

$$5.9. y = \frac{4+3x^3}{x^3\sqrt{(2+x^3)^2}}.$$

$$5.11. y = \frac{x^6 + x^3 - 2}{\sqrt{1+x^3}}.$$

$$5.13. y = \frac{1+x^2}{2\sqrt{1+2x^2}}.$$

$$5.15. y = \frac{\sqrt{(1+x^2)^3}}{3x^3}.$$

$$5.17. y = \frac{\sqrt{2x+3}(x-2)}{x^2}.$$

$$5.2. y = \frac{(2x^2 - 1)\sqrt{1+x^2}}{3x^3}.$$

$$5.4. y = \frac{2x^2 - x - 1}{3\sqrt{2+4x}}.$$

$$5.6. y = \frac{x^2}{2\sqrt{1-3x^4}}.$$

$$5.8. y = \frac{(x^2 - 8)\sqrt{x^2 - 8}}{6x^3}.$$

$$5.10. y = \sqrt[3]{\frac{(1+x^{3/4})^2}{x^{3/2}}}.$$

$$5.12. y = \frac{(x^2 - 2)\sqrt{4+x^2}}{24x^3}.$$

$$5.14. y = \frac{\sqrt{x-1}(3x+2)}{4x^2}.$$

$$5.16. y = \frac{x^6 + 8x^3 - 128}{\sqrt{8-x^3}}.$$

$$5.18. y = (1-x^2)\sqrt[5]{x^3 + \frac{1}{x}}.$$

$$5.19. y = \frac{(2x^2 + 3)\sqrt{x^2 - 3}}{9x^3}.$$

$$5.20. y = \frac{x-1}{(x^2 + 5)\sqrt{x^2 + 5}}.$$

$$5.21. y = \frac{(2x+1)\sqrt{x^2 - x}}{x^2}.$$

$$5.22. y = 2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}.$$

$$5.23. y = \frac{1}{(x+2)\sqrt{x^2 + 4x + 5}}.$$

$$5.24. y = 3\frac{\sqrt[3]{x^2 + x + 1}}{x+1}.$$

$$5.25. y = 3 \cdot \sqrt[3]{\frac{(x+1)}{(x-1)^2}}.$$

$$5.26. y = \frac{x+7}{6\sqrt{x^2 + 2x + 7}}.$$

$$5.27. y = \frac{x\sqrt{x+1}}{x^2 + x + 1}.$$

$$5.28. y = \frac{x^2 + 2}{2\sqrt{1-x^4}}.$$

$$5.29. y = \frac{(x+3)\sqrt{2x-1}}{2x+7}.$$

$$5.30. y = \frac{3x + \sqrt{x}}{\sqrt{x^2 + 2}}.$$

$$5.31. y = \frac{3x^6 + 4x^4 - x^2 - 2}{15\sqrt{1+x^2}}.$$

**Задача 6.** Найти производную.

$$6.1. y = x - \ln(2 + e^x + 2\sqrt{e^{2x} + e^x + 1}).$$

$$6.2. y = e^{2x}(2 - \sin 2x - \cos 2x)/8.$$

$$6.3. y = \frac{1}{2} \operatorname{arctg} \frac{e^x - 3}{2}.$$

$$6.4. y = \frac{1}{\ln 4} \ln \frac{1+2^x}{1-2^x}.$$

$$6.5. y = 2\sqrt{e^x + 1} + \ln \frac{\sqrt{e^x + 1} - 1}{\sqrt{e^x + 1} + 1}.$$

$$6.6. y = \frac{2}{3} \sqrt{(\operatorname{arctg} e^x)^3}.$$

$$6.7. y = \frac{1}{2} \ln(e^{2x} + 1) - 2 \operatorname{arctg} e^x.$$

$$6.8. y = \ln(e^x + 1) + \frac{18e^{2x} + 27e^x + 11}{6(e^x + 1)^3}.$$

$$6.9. y = \frac{2(\sqrt{2^x - 1} - \operatorname{arctg} \sqrt{2^x - 1})}{\ln 2}.$$

$$6.10. y = 2(x - 2)\sqrt{1 + e^x} - 2 \ln \frac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}.$$

$$6.11. y = \frac{e^{\alpha x} (\alpha \sin \beta x - \beta \cos \beta x)}{\alpha^2 + \beta^2}.$$

$$6.12. y = \frac{e^{\alpha x} (\beta \sin \beta x - \alpha \cos \beta x)}{\alpha^2 + \beta^2}.$$

$$6.13. y = e^{\alpha x} \left[ \frac{1}{2a} + \frac{a \cos 2bx + 2b \sin 2bx}{2(a^2 + 4b^2)} \right].$$

$$6.14. y = x + \frac{1}{1 + e^x} - \ln(1 + e^x).$$

$$6.15. y = x - 3 \ln \left[ (1 + e^{x/6}) \sqrt{1 + e^{x/3}} \right] - 3 \operatorname{arctg} e^{x/6}.$$

$$6.16. y = x + \frac{8}{1 + e^{x/4}}.$$

$$6.17. y = \ln(e^x + \sqrt{e^{2x} - 1}) + \arcsin e^{-x}.$$

$$6.18. y = x - e^{-x} \arcsin e^x - \ln(1 + \sqrt{1 - e^{2x}}).$$

$$6.19. y = x - \ln(1 + e^x) - 2e^{-x/2} \operatorname{arctg} e^{x/2} - (\operatorname{arctg} e^{x/2})^2.$$

$$6.20. y = \frac{e^{-x^3}}{1 + x^3}.$$

$$6.21. y = \frac{1}{m\sqrt{ab}} \operatorname{arctg} \left( e^{mx} \cdot \sqrt{\frac{a}{b}} \right).$$

$$6.22. y = 3e^{\sqrt[3]{x}} \left( \sqrt[3]{x^2} - 2\sqrt[3]{x} + 2 \right).$$

$$6.23. y = \ln \frac{\sqrt{1 + e^x + e^{2x}} - e^x - 1}{\sqrt{1 + e^x + e^{2x}} - e^x + 1}.$$

$$6.24. y = e^{\sin x} \left( x - \frac{1}{\cos x} \right).$$

$$6.25. y = \frac{e^x}{2} \left[ (x^2 - 1) \cos x + (x - 1)^2 \sin x \right].$$

$$6.26. y = \operatorname{arctg}(e^x - e^{-x}).$$

$$6.27. y = 3e^{\sqrt[3]{x}} \left( \sqrt[3]{x^5} - 5\sqrt[3]{x^4} + 20x - 60\sqrt[3]{x^2} + 120\sqrt[3]{x} - 120 \right).$$

$$6.28. y = -\frac{e^{3x}}{3\operatorname{sh}^3 x}.$$

$$6.29. y = \arcsin e^{-x} - \sqrt{1 - e^{2x}}.$$

$$6.30. y = -\frac{1}{2}e^{-x^2} (x^4 + 2x^2 + 2).$$

$$6.31. y = \frac{e^{x^2}}{1 + x^2}.$$

**Задача 7.** Найти производную.

$$7.1. y = \sqrt{x} \ln(\sqrt{x} + \sqrt{x+a}) - \sqrt{x+a}.$$

$$7.2. y = \ln(x + \sqrt{a^2 + x^2}).$$

$$7.3. y = 2\sqrt{x} - 4\ln(2 + \sqrt{x}).$$

$$7.4. y = \ln \frac{x^2}{\sqrt{1 - ax^4}}.$$

$$7.5. y = \ln(\sqrt{x} + \sqrt{x+1}).$$

$$7.6. y = \ln \frac{a^2 + x^2}{a^2 - x^2}.$$

$$7.7. y = \ln^2(x + \cos x).$$

$$7.8. y = \ln^3(1 + \cos x).$$

$$7.9. y = \ln \frac{x^2}{1 - x^2}.$$

$$7.10. y = \operatorname{Intg} \left( \frac{\pi}{4} + \frac{x}{2} \right).$$

$$7.11. y = \ln^4 \sqrt{\frac{1+2x}{1-2x}}.$$

$$7.12. y = x + \frac{1}{\sqrt{2}} \ln \frac{x - \sqrt{2}}{x + \sqrt{2}} + a^{\pi\sqrt{2}}.$$

$$7.13. y = \ln \sin \frac{2x+4}{x+1}.$$

$$7.14. y = \log_{16} \log_5 \operatorname{tg} x.$$

$$7.15. y = \log_4 \log_2 \operatorname{tg} x.$$

$$7.16. y = x(\cos \ln x + \sin \ln x)/2.$$

$$7.17. y = \ln \cos \frac{2x+3}{x+1}.$$

$$7.18. y = \lg \ln(\operatorname{ctg} x).$$

$$7.19. y = \log_a \frac{1}{\sqrt{1-x^4}}.$$

$$7.20. y = \frac{1}{\sqrt{2}} \ln(\sqrt{2} \operatorname{tg} x + \sqrt{1+2\operatorname{tg}^2 x}).$$

$$7.21. y = \ln \arcsin \sqrt{1 - e^{2x}}.$$

$$7.22. y = \ln \arccos \sqrt{1 - e^{4x}}.$$

$$7.23. y = \ln\left(bx + \sqrt{a^2 + b^2x^2}\right).$$

$$7.25. y = \ln\left(\arccos\frac{1}{\sqrt{x}}\right).$$

$$7.27. y = \ln\frac{\sqrt{5} + \operatorname{tg}(x/2)}{\sqrt{5} - \operatorname{tg}(x/2)}.$$

$$7.29. y = \ln \ln \sin(1 + 1/x).$$

$$7.31. y = \ln \ln^2 \ln^3 x.$$

$$7.24. y = \ln\frac{\sqrt{x^2 + 1} + x\sqrt{2}}{\sqrt{x^2 + 1} - x\sqrt{2}}.$$

$$7.26. y = \ln\left(e^x + \sqrt{1 + e^{2x}}\right).$$

$$7.28. y = \ln\frac{\ln x}{\sin(1/x)}.$$

$$7.30. y = \ln \ln^3 \ln^2 x.$$

**Задача 8.** Найти производную.

$$8.1. y = \sin\sqrt{3} + \frac{1 \sin^2 3x}{3 \cos 6x}.$$

$$8.3. y = \operatorname{tg} \lg \frac{1}{3} + \frac{1 \sin^2 4x}{4 \cos 8x}.$$

$$8.5. y = \frac{\cos \sin 5 \cdot \sin^2 2x}{2 \cos 4x}.$$

$$8.7. y = \frac{\cos \ln 7 \cdot \sin^2 7x}{7 \cos 14x}.$$

$$8.9. y = \operatorname{ctg}(\cos 2) + \frac{1 \sin^2 6x}{6 \cos 12x}.$$

$$8.11. y = \frac{1}{3} \cos\left(\operatorname{tg} \frac{1}{2}\right) + \frac{1 \sin^2 10x}{10 \cos 20x}.$$

$$8.13. y = 8 \sin(\operatorname{ctg} 3) + \frac{1 \sin^2 5x}{5 \cos 10x}.$$

$$8.15. y = \frac{\cos\left(\operatorname{tg} \frac{1}{3}\right) \cdot \sin^2 15x}{15 \cos 30x}.$$

$$8.2. y = \cos \ln 2 - \frac{1 \cos^2 3x}{3 \sin 6x}.$$

$$8.4. y = \operatorname{ctg} \sqrt[3]{5} - \frac{1 \cos^2 4x}{8 \sin 8x}.$$

$$8.6. y = \frac{\sin \cos 3 \cdot \cos^2 2x}{4 \sin 4x}.$$

$$8.8. y = \cos(\operatorname{ctg} 2) - \frac{1 \cos^2 8x}{16 \sin 16x}.$$

$$8.10. y = \sqrt[3]{\operatorname{ctg} 2} - \frac{1 \cos^2 10x}{20 \sin 20x}.$$

$$8.12. y = \ln \sin \frac{1}{2} - \frac{1 \cos^2 12x}{24 \sin 24x}.$$

$$8.14. y = \frac{\cos(\operatorname{ctg} 3) \cdot \cos^2 14x}{28 \sin 28x}.$$

$$8.16. y = \frac{\sin\left(\operatorname{tg} \frac{1}{7}\right) \cdot \cos^2 16x}{32 \sin 32x}.$$

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$$8.17. y = \frac{\operatorname{ctg}\left(\sin\frac{1}{3}\right) \cdot \sin^2 17x}{17 \cos 34x}.$$

$$8.19. y = \frac{\operatorname{tg}(\ln 2) \cdot \sin^2 19x}{19 \cos 38x}.$$

$$8.21. y = \sqrt{\operatorname{tg} 4} + \frac{\sin^2 21x}{21 \cos 42x}.$$

$$8.23. y = \ln \cos \frac{1}{3} + \frac{\sin^2 23x}{23 \cos 46x}.$$

$$8.25. y = \sin \ln 2 + \frac{\sin^2 25x}{25 \cos 50x}.$$

$$8.27. y = \sqrt[7]{\operatorname{tg}(\cos 2)} + \frac{\sin^2 27x}{27 \cos 54x}.$$

$$8.29. y = \cos^2 \sin 3 + \frac{\sin^2 29x}{29 \cos 58x}.$$

$$8.31. y = \operatorname{tg} \sqrt{\cos(1/3)} + \frac{\sin^2 31x}{31 \cos 62x}.$$

$$8.18. y = \frac{\sqrt[5]{\operatorname{ctg} 2} \cdot \cos^2 18x}{36 \sin 36x}.$$

$$8.20. y = \operatorname{ctg}(\cos 5) - \frac{1 \cos^2 20x}{40 \sin 40x}.$$

$$8.22. y = \cos(\ln 13) - \frac{1 \cos^2 22x}{44 \sin 44x}.$$

$$8.24. y = \operatorname{ctg}\left(\sin\frac{1}{13}\right) - \frac{1 \cos^2 24x}{48 \sin 48x}.$$

$$8.26. y = \sqrt[3]{\cos \sqrt{2}} - \frac{1 \cos^2 26x}{52 \sin 52x}.$$

$$8.28. y = \sin \sqrt[3]{\operatorname{tg} 2} - \frac{\cos^2 28x}{56 \sin 56x}.$$

$$8.30. y = \sin^3 \cos 2 - \frac{\cos^2 30x}{60 \sin 60x}.$$

**Задача 9.** Найти производную.

$$9.1. y = \operatorname{arctg} \frac{\operatorname{tg} x - \operatorname{ctg} x}{\sqrt{2}}.$$

$$9.3. y = \frac{2x-1}{4} \sqrt{2+x-x^2} + \frac{9}{8} \operatorname{arcsin} \frac{2x-1}{3}.$$

$$9.5. y = \arccos \frac{x^2 - 4}{\sqrt{x^4 + 16}}.$$

$$9.7. y = \frac{1}{4} \ln \frac{x-1}{x+1} - \frac{1}{2} \operatorname{arctg} x.$$

$$9.2. y = \operatorname{arcsin} \frac{\sqrt{x} - 2}{\sqrt{5x}}.$$

$$9.4. y = \operatorname{arctg} \frac{\sqrt{1+x^2} - 1}{x}.$$

$$9.6. y = \sqrt{\frac{2}{3}} \operatorname{arctg} \frac{3x-1}{\sqrt{6x}}.$$

$$9.8. y = \frac{1}{2}(x-4)\sqrt{8x-x^2-7} - 9\arccos\sqrt{\frac{x-1}{6}}.$$

$$9.9. y = \frac{(1+x)\operatorname{arctg}\sqrt{x}}{x^2} + \frac{1}{3x\sqrt{x}}.$$

$$9.10. y = \frac{x^3}{3}\arccos x - \frac{2+x^2}{9}\sqrt{1-x^2}.$$

$$9.11. y = \frac{1}{2\sqrt{x}} + \frac{1+x}{2x}\operatorname{arctg}\sqrt{x}.$$

$$9.12. y = \frac{3+x}{2}\sqrt{x(2-x)} + 3\arccos\sqrt{\frac{x}{2}}.$$

$$9.13. y = \frac{4+x^4}{x^3}\operatorname{arctg}\frac{x^2}{2} + \frac{4}{x}.$$

$$9.14. y = \arcsin\sqrt{\frac{x}{x+1}} + \operatorname{arctg}\sqrt{x}.$$

$$9.15. y = \frac{1}{2}\sqrt{\frac{1}{x^2}-1} - \frac{\arccos x}{2x^2}.$$

$$9.16. y = 6\arcsin\frac{\sqrt{x}}{2} - \frac{6+x}{2}\sqrt{x(4-x)}.$$

$$9.17. y = \frac{x-3}{2}\sqrt{6x-x^2-8} + \arcsin\sqrt{\frac{x}{2}-1}.$$

$$9.18. y = \frac{(1+x)\operatorname{arctg}\sqrt{x}-\sqrt{x}}{x}.$$

$$9.19. y = \frac{2\sqrt{1-x}\arcsin\sqrt{x}}{x} + \frac{2}{\sqrt{x}}.$$

$$9.20. y = \frac{2x-5}{4}\sqrt{5x-4-x^2} + \frac{9}{4}\arcsin\sqrt{\frac{x-1}{3}}.$$

$$9.21. y = \operatorname{arctg} x + \frac{5}{6}\ln\frac{x^2+1}{x^2+4}.$$

$$9.22. y = \arcsin\frac{x-2}{(x-1)\sqrt{2}}.$$

$$9.23. y = \sqrt{1-x^2} - x\arcsin\sqrt{1-x^2}.$$

$$9.24. y = \sqrt{x} + \frac{1}{3}\operatorname{arctg}\sqrt{x} + \frac{8}{3}\operatorname{arctg}\frac{\sqrt{x}}{2}.$$

$$9.25. y = \operatorname{arctg}\frac{\sqrt{1-x}}{1-\sqrt{x}}.$$

$$9.26. y = (2x^2+6x+5)\operatorname{arctg}\frac{x+1}{x+2} - x.$$

$$9.27. y = \frac{x}{2\sqrt{1-4x^2}}\arcsin 2x + \frac{1}{8}\ln(1-4x^2).$$

$$9.28. y = \left(2x^2 - x + \frac{1}{2}\right)\operatorname{arctg}\frac{x^2-1}{x\sqrt{3}} - \frac{x^3}{2\sqrt{3}} - \frac{\sqrt{3}}{2}x.$$

$$9.29. y = (x + 2\sqrt{x} + 2) \operatorname{arctg} \frac{\sqrt{x}}{\sqrt{x} + 2} - \sqrt{x}.$$

$$9.30. y = \sqrt{1 + 2x - x^2} \arcsin \frac{x\sqrt{2}}{1+x} - \sqrt{2} \ln(1+x).$$

$$9.31. y = \operatorname{arctg} \frac{\operatorname{tg}(x/2) + 1}{2}.$$

**Задача 10.** Найти производную.

$$10.1. y = \frac{1}{4\sqrt{5}} \ln \frac{2 + \sqrt{5} \operatorname{th} x}{2 - \sqrt{5} \operatorname{th} x}.$$

$$10.2. y = \frac{\operatorname{sh} x}{4 \operatorname{ch}^4 x} + \frac{3 \operatorname{sh} x}{8 \operatorname{ch}^2 x} + \frac{3}{8} \operatorname{arctg}(\operatorname{sh} x).$$

$$10.3. y = \frac{1}{2} \ln \frac{1 + \sqrt{\operatorname{th} x}}{1 - \sqrt{\operatorname{th} x}} - \operatorname{arctg} \sqrt{\operatorname{th} x}.$$

$$10.4. y = \frac{3}{8\sqrt{2}} \ln \frac{\sqrt{2} + \operatorname{th} x}{\sqrt{2} - \operatorname{th} x} - \frac{\operatorname{th} x}{4(2 - \operatorname{th}^2 x)}.$$

$$10.5. y = \frac{1}{2} \operatorname{th} x + \frac{1}{4\sqrt{2}} \ln \frac{1 + \sqrt{2} \operatorname{th} x}{1 - \sqrt{2} \operatorname{th} x}.$$

$$10.6. y = -\frac{1}{2} \ln \left( \operatorname{th} \frac{x}{2} \right) - \frac{\operatorname{ch} x}{2 \operatorname{sh}^2 x}.$$

$$10.7. y = \frac{1}{2a\sqrt{1+a^2}} \ln \frac{a + \sqrt{1+a^2} \operatorname{th} x}{a - \sqrt{1+a^2} \operatorname{th} x}.$$

$$10.8. y = \frac{1}{18\sqrt{2}} \ln \frac{1 + \sqrt{2} \operatorname{cth} x}{1 - \sqrt{2} \operatorname{cth} x}.$$

$$10.9. y = \operatorname{arctg} \frac{\sqrt{\operatorname{sh} 2x}}{\operatorname{ch} x - \operatorname{sh} x}.$$

$$10.10. y = \frac{1}{6} \ln \frac{1 - \operatorname{sh} 2x}{2 + \operatorname{sh} 2x}.$$

$$10.11. y = \sqrt[4]{\frac{1 + \operatorname{th} x}{1 - \operatorname{th} x}}.$$

$$10.12. y = \frac{\operatorname{sh} x}{1 + \operatorname{ch} x}.$$

$$10.13. y = \frac{\operatorname{ch} x}{\sqrt{\operatorname{sh} 2x}}.$$

$$10.14. y = \frac{\operatorname{sh} 3x}{\sqrt{\operatorname{ch} 6x}}.$$

$$10.15. y = \frac{1 + 8 \operatorname{ch}^2 x \cdot \ln(\operatorname{ch} x)}{2 \operatorname{ch}^2 x}.$$

$$10.16. y = -\frac{12 \operatorname{sh}^2 x + 1}{3 \operatorname{sh}^2 x}.$$

$$10.17. y = -\frac{\operatorname{sh} x}{2 \operatorname{ch}^2 x} + \frac{3}{2} \arcsin(\operatorname{th} x).$$

$$10.18. y = \frac{1}{\sqrt{8}} \arcsin \frac{3 + \operatorname{ch} x}{1 + 3 \operatorname{ch} x}.$$



$$10.19. y = \frac{1}{\sqrt{8}} \ln \frac{4 + \sqrt{8} \operatorname{th} \frac{x}{2}}{4 - \sqrt{8} \operatorname{th} \frac{x}{2}}.$$

$$10.20. y = \frac{1}{4} \ln \left| \operatorname{th} \frac{x}{2} \right| - \frac{1}{4} \ln \frac{3 + \operatorname{ch} x}{\operatorname{sh} x}.$$

$$10.21. y = -\frac{1}{4} \arcsin \frac{5 + 3 \operatorname{ch} x}{3 + 5 \operatorname{ch} x}.$$

$$10.22. y = \frac{1 - 8 \operatorname{ch}^2 x}{4 \operatorname{ch}^4 x}.$$

$$10.23. y = \frac{2}{\operatorname{sh} x} - \frac{1}{3 \operatorname{sh}^3 x} + \frac{\operatorname{sh} x}{2 \operatorname{ch}^2 x} + \frac{5}{2} \operatorname{arctg}(\operatorname{sh} x).$$

$$10.24. y = \frac{8}{3} \operatorname{cth} 2x - \frac{1}{3 \operatorname{ch} x \cdot \operatorname{sh}^3 x}.$$

$$10.25. y = \frac{1}{2} \operatorname{arctg}(\operatorname{sh} x) - \frac{\operatorname{sh} x}{2 \operatorname{ch}^2 x}.$$

$$10.26. y = \frac{3}{2} \ln \left( \operatorname{th} \frac{x}{2} \right) + \operatorname{ch} x - \frac{\operatorname{ch} x}{2 \operatorname{sh}^2 x}.$$

$$10.27. y = -\frac{\operatorname{sh} x}{2 \operatorname{ch}^2 x} - \frac{1}{\operatorname{sh} x} - \frac{3}{2} \operatorname{arctg}(\operatorname{sh} x).$$

$$10.28. y = \frac{\operatorname{sh} x}{2 \operatorname{ch}^2 x} + \frac{1}{2} \operatorname{arctg}(\operatorname{sh} x).$$

$$10.29. y = \frac{1}{2} \left[ \frac{\operatorname{sh} x}{\operatorname{ch}^2 x} + \operatorname{arctg}(\operatorname{sh} x) \right].$$

$$10.30. y = -\frac{\operatorname{ch} x}{2 \operatorname{sh}^2 x} - \frac{1}{2} \ln \left( \operatorname{th} \frac{x}{2} \right).$$

$$10.31. y = \frac{2}{3} \operatorname{cth} x - \frac{\operatorname{ch} x}{3 \operatorname{sh}^3 x}.$$

**Задача 11.** Найти производную.

$$11.1. y = (\operatorname{arctg} x)^{(1/2) \ln(\operatorname{arctg} x)}.$$

$$11.2. y = (\sin \sqrt{x})^{\ln(\sin \sqrt{x})}.$$

$$11.3. y = (\sin x)^{5e^x}.$$

$$11.4. y = (\arcsin x)^{e^x}.$$

$$11.5. y = (\ln x)^{3^x}.$$

$$11.6. y = x^{\arcsin x}.$$

$$11.7. y = (\operatorname{ctg} 3x)^{2e^x}.$$

$$11.8. y = x^{e^{\operatorname{tg} x}}.$$

$$11.9. y = (\operatorname{tg} x)^{4e^x}.$$

$$11.10. y = (\cos 5x)^{e^x}.$$

11.11.  $y = (x \sin x)^{8 \ln(x \sin x)}$ .

11.12.  $y = (x - 5)^{\operatorname{ch} x}$ .

11.13.  $y = (x^3 + 4)^{\operatorname{tg} x}$ .

11.14.  $y = x^{\sin x^3}$ .

11.15.  $y = (x^2 - 1)^{\operatorname{sh} x}$ .

11.16.  $y = (x^4 + 5)^{\operatorname{ctg} x}$ .

11.17.  $y = (\sin x)^{5x/2}$ .

11.18.  $y = (x^2 + 1)^{\cos x}$ .

11.19.  $y = 19^{x^{19}} x^{19}$ .

11.20.  $y = x^{3^x} \cdot 2^x$ .

11.21.  $y = (\sin \sqrt{x})^{e^{1/x}}$ .

11.22.  $y = x^{e^{\operatorname{ctg} x}}$ .

11.23.  $y = x^{e^{\cos x}}$ .

11.24.  $y = x^{2^x} \cdot 5^x$ .

11.25.  $y = x^{e^{\sin x}}$ .

11.26.  $y = (\operatorname{tg} x)^{\ln(\operatorname{tg} x)/4}$ .

11.27.  $y = x^{e^{\operatorname{arctg} x}}$ .

11.28.  $y = (x^8 + 1)^{\operatorname{th} x}$ .

11.29.  $y = x^{29^x} \cdot 29^x$ .

11.30.  $y = (\cos 2x)^{\ln(\cos 2x)/4}$ .

11.31.  $y = x^{e^x} x^9$ .

**Задача 12.** Найти производную.

12.1.  $y = \frac{1}{24}(x^2 + 8)\sqrt{x^2 - 4} + \frac{x^2}{16} \arcsin \frac{2}{x}, \quad x > 0.$

12.2.  $y = \frac{4x + 1}{16x^2 + 8x + 3} + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{4x + 1}{\sqrt{2}}.$

12.3.  $y = 2x - \ln(1 + \sqrt{1 - e^{4x}}) - e^{-2x} \arcsin(e^{2x}).$

12.4.  $y = \sqrt{9x^2 - 12x + 5} \operatorname{arctg}(3x - 2) - \ln(3x - 2 + \sqrt{9x^2 - 12x + 5}).$

12.5.  $y = \frac{2}{x-1} \sqrt{2x-x^2} + \ln \frac{1 + \sqrt{2x-x^2}}{x-1}.$

$$12.6. y = \frac{x^2}{81} \arcsin \frac{3}{x} + \frac{1}{81} (x^2 + 18) \sqrt{x^2 - 9}, \quad x > 0.$$

$$12.7. y = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{3x-1}{\sqrt{2}} + \frac{1}{3} \cdot \frac{3x-1}{3x^2-2x+1}.$$

$$12.8. y = 3x - \ln(1 + \sqrt{1 - e^{6x}}) - e^{-3x} \arcsin(e^{3x}).$$

$$12.9. y = \ln(4x - 1 + \sqrt{16x^2 - 8x + 2}) - \sqrt{16x^2 - 8x + 2} \operatorname{arctg}(4x - 1).$$

$$12.10. y = \ln \frac{1 + 2\sqrt{-x - x^2}}{2x + 1} + \frac{4}{2x + 1} \sqrt{-x - x^2}.$$

$$12.11. y = (2x + 3)^4 \cdot \arcsin \frac{1}{2x + 3} + \frac{2}{3} (4x^2 + 12x + 11) \sqrt{x^2 + 3x + 2}, \quad 2x + 3 > 0.$$

$$12.12. y = \frac{x + 2}{x^2 + 4x + 6} + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x + 2}{\sqrt{2}}.$$

$$12.13. y = 5x - \ln(1 + \sqrt{1 - e^{10x}}) - e^{-5x} \arcsin(e^{5x}).$$

$$12.14. y = \sqrt{x^2 - 8x + 17} \operatorname{arctg}(x - 4) - \ln(x - 4 + \sqrt{x^2 - 8x + 17}).$$

$$12.15. y = \ln \frac{1 + \sqrt{-3 + 4x - x^2}}{2 - x} + \frac{2}{2 - x} \sqrt{-3 + 4x - x^2}.$$

$$12.16. y = (3x^2 - 4x + 2) \sqrt{9x^2 - 12x + 3} + (3x - 2)^4 \arcsin \frac{1}{3x - 2}, \quad 3x - 2 > 0.$$

$$12.17. y = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x-1}{\sqrt{2}} + \frac{x-1}{x^2-2x+3}.$$

$$12.18. y = \ln(e^{5x} + \sqrt{e^{10x} - 1}) + \arcsin(e^{-5x}).$$

$$12.19. y = \ln(2x - 3 + \sqrt{4x^2 - 12x + 10}) - \sqrt{4x^2 - 12x + 10} \operatorname{arctg}(2x - 3).$$

$$12.20. y = \ln \frac{1 + \sqrt{-3 - 4x - x^2}}{-x - 2} - \frac{2}{x + 2} \sqrt{-3 - 4x - x^2}.$$

$$12.21. y = \frac{2}{3}(4x^2 - 4x + 3)\sqrt{x^2 - x} + (2x - 1)^4 \arcsin \frac{1}{2x - 1}, \quad 2x - 1 > 0.$$

$$12.22. y = \frac{2x - 1}{4x^2 - 4x + 3} + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{2x - 1}{\sqrt{2}}.$$

$$12.23. y = \arcsin(e^{-4x}) + \ln(e^{4x} + \sqrt{e^{8x} - 1}).$$

$$12.24. y = \ln(5x + \sqrt{25x^2 + 1}) - \sqrt{25x^2 + 1} \operatorname{arctg} 5x.$$

$$12.25. y = \frac{2}{3x - 2} \sqrt{-3 + 12x - 9x^2} + \ln \frac{1 + \sqrt{-3 + 12x - 9x^2}}{3x - 2}.$$

$$12.26. y = (3x + 1)^4 \arcsin \frac{1}{3x + 1} + (3x^2 + 2x + 1)\sqrt{9x^2 + 6x}, \quad 3x + 1 > 0.$$

$$12.27. y = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{2x + 1}{\sqrt{2}} + \frac{2x + 1}{4x^2 + 4x + 3}.$$

$$12.28. y = \ln(e^{3x} + \sqrt{e^{6x} - 1}) + \arcsin(e^{-3x}).$$

$$12.29. y = \sqrt{49x^2 + 1} \operatorname{arctg} 7x - \ln(7x + \sqrt{49x^2 + 1}).$$

$$12.30. y = \frac{1}{x} \sqrt{1 - 4x^2} + \ln \frac{1 + \sqrt{1 + 4x^2}}{2x}.$$

$$12.31. y = \arcsin(e^{-2x}) + \ln(e^{2x} + \sqrt{e^{4x} - 1}).$$

**Задача 13.** Найти производную.

$$13.1. y = \frac{x \arcsin x}{\sqrt{1 - x^2}} + \ln \sqrt{1 - x^2}.$$

$$13.2. y = 4 \ln \frac{x}{1 + \sqrt{1 - 4x^2}} - \frac{\sqrt{1 - 4x^2}}{x^2}.$$

$$13.3. y = x(2x^2 + 5)\sqrt{x^2 + 1} + 3 \ln(x + \sqrt{x^2 + 1}).$$

$$13.4. y = x^3 \arcsin x + \frac{x^2 + 2}{3} \sqrt{1 - x^2}.$$

$$13.5. y = 3 \arcsin \frac{3}{4x+1} + 2\sqrt{4x^2 + 2x - 2}, \quad 4x+1 > 0.$$

$$13.6. y = \sqrt{1+x^2} \operatorname{arctg} x - \ln(x + \sqrt{1+x^2}).$$

$$13.7. y = 2 \arcsin \frac{2}{3x+4} + \sqrt{9x^2 + 24x + 12}, \quad 3x+4 > 0.$$

$$13.8. y = x(2x^2 + 1)\sqrt{x^2 + 1} - \ln(x + \sqrt{x^2 + 1}).$$

$$13.9. y = \ln(x + \sqrt{x^2 + 1}) - \frac{\sqrt{1+x^2}}{x}.$$

$$13.10. y = \sqrt{1-3x-2x^2} + \frac{3}{2\sqrt{2}} \arcsin \frac{4x+3}{\sqrt{17}}.$$

$$13.11. y = \sqrt{(4+x)(1+x)} + 3 \ln(\sqrt{4+x} + \sqrt{1+x}).$$

$$13.12. y = \ln \frac{\sqrt{x^2-x+1}}{x} + \sqrt{3} \operatorname{arctg} \frac{2x-1}{\sqrt{3}}.$$

$$13.13. y = \frac{1}{12} \ln \frac{x^4 - x^2 + 1}{(x^2 + 1)^2} - \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{\sqrt{3}}{2x^2 - 1}.$$

$$13.14. y = 4 \arcsin \frac{4}{2x+3} + \sqrt{4x^2 + 12x - 7}, \quad 2x+3 > 0.$$

$$13.15. y = 2 \arcsin \frac{2}{3x+1} + \sqrt{9x^2 + 6x - 3}, \quad 3x+1 > 0.$$

$$13.16. y = (2+3x)\sqrt{x-1} - \frac{3}{2} \operatorname{arctg} \sqrt{x-1}.$$

$$13.17. y = \frac{1}{3}(x-2)\sqrt{x+1} + \ln(\sqrt{x+1} + 1).$$

$$13.18. y = \sqrt{x^2 + 1} - \frac{1}{2} \ln \frac{\sqrt{x^2 + 1} - x}{\sqrt{x^2 + 1} + 1}.$$

$$13.19. y = \ln \sqrt[3]{\frac{x-1}{x+1}} - \frac{1}{2} \left( \frac{1}{2} + \frac{1}{x^2 - 1} \right) \operatorname{arctg} x.$$

$$13.20. y = x \ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{2}(\arcsin x - x).$$

$$13.21. y = \operatorname{arctg} \sqrt{x^2 - 1} - \frac{\ln x}{\sqrt{x^2 - 1}}. \quad 13.22. y = 3 \arcsin \frac{3}{x+2} + \sqrt{x^2 + 4x - 5}.$$

$$13.23. y = \sqrt{(3-x)(2+x)} + 5 \arcsin \sqrt{\frac{x+2}{5}}.$$

$$13.24. y = x(\arcsin x)^2 + 2\sqrt{1-x^2} \arcsin x - 2x.$$

$$13.25. y = \frac{\sqrt{1-x^2}}{x} + \arcsin x.$$

$$13.26. y = x^2 \arccos x - \frac{x^2+2}{3} \sqrt{1-x^2}.$$

$$13.27. y = \frac{\sqrt{x^2+2}}{x^2} - \frac{1}{\sqrt{2}} \ln \frac{\sqrt{2} + \sqrt{x^2+2}}{x}.$$

$$13.28. y = \frac{x}{4}(10-x^2)\sqrt{4-x^2} + 6 \arcsin \frac{x}{2}.$$

$$13.29. y = \arcsin \frac{1}{2x+3} + 2\sqrt{x^2+3x+2}, \quad 2x+3 > 0.$$

$$13.30. y = x \arcsin \sqrt{\frac{x}{x+1}} - \sqrt{x} + \operatorname{arctg} \sqrt{x}.$$

$$13.31. y = \frac{\arcsin x}{\sqrt{1-x^2}} + \frac{1}{2} \ln \frac{1-x}{1+x}.$$

**Задача 14.** Найти производную.

$$14.1. y = \frac{1}{\sin \alpha} \ln(\operatorname{tg} x + \operatorname{ctg} \alpha).$$

$$14.2. y = x \cos \alpha + \sin \alpha \ln \sin(x - \alpha).$$

$$14.3. y = \frac{1}{2\sqrt{2}} \left[ \sin \ln x - (\sqrt{2} - 1) \cdot \cos \ln x \right] x^{\sqrt{2}+1}.$$

$$14.4. y = \operatorname{arctg} \left( \frac{\cos x}{\sqrt[4]{\cos 2x}} \right).$$

$$14.5. y = 3 \frac{\sin x}{\cos^2 x} + 2 \frac{\sin x}{\cos^4 x}. \quad 14.6. y = (a^2 + b^2)^{-1/2} \cdot \arcsin \left( \frac{\sqrt{a^2 + b^2} \sin x}{b} \right).$$

$$14.7. y = \frac{7^x (3 \sin 3x + \cos 3x \cdot \ln 7)}{9 + \ln^2 7}. \quad 14.8. y = \ln \frac{\sin x}{\cos x + \sqrt{\cos 2x}}.$$

$$14.9. y = \frac{1}{a(1+a^2)} \left[ \operatorname{arctg}(a \cos x) + a \operatorname{Intg} \frac{x}{2} \right].$$

$$14.10. y = -\frac{1}{3 \sin^3 x} - \frac{1}{\sin x} + \frac{1}{2} \ln \frac{1 + \sin x}{1 - \sin x}.$$

$$14.11. y = (1 + x^2) e^{\operatorname{arctg} x}.$$

$$14.12. y = \frac{\operatorname{ctg} x + x}{1 - x \operatorname{ctg} x}.$$

$$14.13. y = \frac{1}{2 \sin \frac{\alpha}{2}} \operatorname{arctg} \frac{2x \sin \frac{\alpha}{2}}{1 - x^2}.$$

$$14.14. y = \operatorname{arctg} \frac{\sqrt{\sqrt{x^4 + 1} - x^2}}{x}, \quad x > 0.$$

$$14.15. y = \frac{6^x (\sin 4x \cdot \ln 6 - 4 \cos 4x)}{16 + \ln^2 6}.$$

$$14.16. y = \operatorname{arctg} \frac{\sqrt{2 \operatorname{tg} x}}{1 - \operatorname{tg} x}.$$

$$14.17. y = \operatorname{arctg} \frac{2 \sin x}{\sqrt{9 \cos^2 x - 4}}.$$

$$14.18. y = \frac{5^x (2 \sin 2x + \cos 2x \cdot \ln 5)}{4 + \ln^2 5}.$$

$$14.19. y = \ln \frac{\sqrt{2} + \operatorname{th} x}{\sqrt{2} - \operatorname{th} x}.$$

$$14.20. y = \frac{3^x (4 \sin 4x + \ln 3 \cdot \cos 4x)}{16 + \ln^2 3}.$$

$$14.21. y = \frac{4^x (\ln 4 \cdot \sin 4x - 4 \cos 4x)}{16 + \ln^2 4}.$$

$$14.22. y = \frac{\cos x}{\sin^2 x} - 2 \cos x - 3 \operatorname{Intg} \frac{x}{2}.$$

$$14.23. y = \frac{5^x (\sin 3x \cdot \ln 5 - 3 \cos 3x)}{9 + \ln^2 5}.$$

$$14.24. y = x - \ln(1 + e^x) - 2e^{-\frac{x}{2}} \operatorname{arctg} e^{\frac{x}{2}}.$$

$$14.25. y = \frac{2^x (\sin x + \cos x \cdot \ln 2)}{1 + \ln^2 2}.$$

$$14.26. y = \frac{\ln(\operatorname{ctg} x + \operatorname{ctg} \alpha)}{\sin \alpha}.$$

$$14.27. y = 2 \frac{\cos x}{\sin^4 x} + 3 \frac{\cos x}{\sin^2 x}.$$

$$14.28. y = \frac{\cos x}{3(2 + \sin x)} + \frac{4}{3\sqrt{3}} \operatorname{arctg} \frac{2 \operatorname{tg}(x/2) + 1}{\sqrt{3}}.$$

$$14.29. y = \frac{3^x (\ln 3 \cdot \sin 2x - 2 \cos 2x)}{\ln^2 3 + 4}. \quad 14.30. y = \frac{1}{2} \ln \frac{1 + \cos x}{1 - \cos x} - \frac{1}{\cos x} - \frac{1}{3 \cos^3 x}.$$

$$14.31. y = \sqrt{\frac{\operatorname{tg} x + \sqrt{2 \operatorname{tg} x + 1}}{\operatorname{tg} x - \sqrt{2 \operatorname{tg} x + 1}}}.$$

**Задача 15.** Найти производную  $y'_x$ .

$$15.1. \begin{cases} x = \frac{3t^2 + 1}{3t^3}, \\ y = \sin\left(\frac{t^3}{3} + t\right). \end{cases}$$

$$15.2. \begin{cases} x = \sqrt{1-t^2}, \\ y = \operatorname{tg} \sqrt{1+t}. \end{cases}$$

$$15.3. \begin{cases} x = \sqrt{2t-t^2}, \\ y = \frac{1}{\sqrt[3]{(1-t)^2}}. \end{cases}$$

$$15.4. \begin{cases} x = \arcsin(\sin t), \\ y = \arccos(\cos t). \end{cases}$$

$$15.5. \begin{cases} x = \ln(t + \sqrt{t^2 + 1}), \\ y = t\sqrt{t^2 + 1}. \end{cases}$$

$$15.6. \begin{cases} x = \sqrt{2t-t^2}, \\ y = \arcsin(t-1). \end{cases}$$

$$15.7. \begin{cases} x = \operatorname{ctg}(2e^t), \\ y = \ln(\operatorname{tge}^t). \end{cases}$$

$$15.8. \begin{cases} x = \ln(\operatorname{ctg} t), \\ y = \frac{1}{\cos^2 t}. \end{cases}$$

$$15.9. \begin{cases} x = \operatorname{arctge}^{t/2}, \\ y = \sqrt{e^t + 1}. \end{cases}$$

$$15.10. \begin{cases} x = \ln \sqrt{\frac{1-t}{1+t}}, \\ y = \sqrt{1-t^2}. \end{cases}$$



$$15.11. \begin{cases} x = \ln \frac{1}{\sqrt{1-t^4}}, \\ y = \arcsin \frac{1-t^2}{1+t^2}. \end{cases}$$

$$15.12. \begin{cases} x = \sqrt{1-t^2}, \\ y = \frac{t}{\sqrt{1-t^2}}. \end{cases}$$

$$15.13. \begin{cases} x = \arcsin(\sqrt{1-t^2}), \\ y = (\arccost)^2. \end{cases}$$

$$15.14. \begin{cases} x = \frac{t}{\sqrt{1-t^2}}, \\ y = \ln \frac{1+\sqrt{1-t^2}}{t}. \end{cases}$$

$$15.15. \begin{cases} x = (1 + \cos^2 t)^2, \\ y = \frac{\cos t}{\sin^2 t}. \end{cases}$$

$$15.16. \begin{cases} x = \ln \frac{1-t}{1+t}, \\ y = \sqrt{1-t^2}. \end{cases}$$

$$15.17. \begin{cases} x = \arccos \frac{1}{t}, \\ y = \sqrt{t^2-1} + \arcsin \frac{1}{t}. \end{cases}$$

$$15.18. \begin{cases} x = \frac{1}{\ln t}, \\ y = \ln \frac{1+\sqrt{1-t^2}}{t}. \end{cases}$$

$$15.19. \begin{cases} x = \arcsin \sqrt{t}, \\ y = \sqrt{1+\sqrt{t}}. \end{cases}$$

$$15.20. \begin{cases} x = (\arcsin t)^2, \\ y = \frac{t}{\sqrt{1-t^2}}. \end{cases}$$

$$15.21. \begin{cases} x = t\sqrt{t^2+1}, \\ y = \ln \frac{1+\sqrt{1+t^2}}{t}. \end{cases}$$

$$15.22. \begin{cases} x = \operatorname{arctg} t, \\ y = \ln \frac{\sqrt{1+t^2}}{t+1}. \end{cases}$$

$$15.23. \begin{cases} x = \ln(1-t^2), \\ y = \arcsin \sqrt{1-t^2}. \end{cases}$$

$$15.24. \begin{cases} x = \operatorname{arctg} \frac{t+1}{t-1}, \\ y = \arcsin \sqrt{1-t^2}. \end{cases}$$

$$15.25. \begin{cases} x = \ln \sqrt{\frac{1-\sin t}{1+\sin t}}, \\ y = \frac{1}{2} \operatorname{tg}^2 t + \ln \cos t. \end{cases}$$

$$15.26. \begin{cases} x = \sqrt{t-t^2} - \operatorname{arctg} \sqrt{\frac{1-t}{t}}, \\ y = \sqrt{t} - \sqrt{1-t} \arcsin \sqrt{t}. \end{cases}$$

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$$15.27. \begin{cases} x = \operatorname{Intg} t, \\ y = \frac{1}{\sin^2 t}. \end{cases}$$

$$15.58. \begin{cases} x = \frac{t^2 \ln t}{1-t^2} + \ln \sqrt{1-t^2}, \\ y = \frac{t}{\sqrt{1-t^2}} \arcsin t + \ln \sqrt{1-t^2}. \end{cases}$$

$$15.29. \begin{cases} x = e^{\sec^2 t}, \\ y = \operatorname{tg} t \cdot \ln \operatorname{cost} + \operatorname{tg} t - t. \end{cases}$$

$$15.30. \begin{cases} x = \frac{t}{\sqrt{1-t^2}} \arcsin t + \ln \sqrt{1-t^2}, \\ y = \frac{t}{\sqrt{1-t^2}}. \end{cases}$$

$$15.31. \begin{cases} x = \ln(t + \sqrt{1+t^2}), \\ y = \sqrt{1+t^2} - \ln \frac{1 + \sqrt{1+t^2}}{t}. \end{cases}$$

**Задача 16.** Составить уравнения касательной и нормали к кривой в точке, соответствующей значению параметра  $t = t_0$ .

$$16.1. \begin{cases} x = a \sin^3 t, \\ y = a \cos^3 t, \quad t_0 = \pi/3. \end{cases}$$

$$16.2. \begin{cases} x = \sqrt{3} \operatorname{cost}, \\ y = \sin t, \quad t_0 = \pi/3. \end{cases}$$

$$16.3. \begin{cases} x = a(t - \sin t), \\ y = a(1 - \operatorname{cost}), \quad t_0 = \pi/3. \end{cases}$$

$$16.4. \begin{cases} x = 2t - t^2, \\ y = 3t - t^3, \quad t_0 = 1. \end{cases}$$

$$16.5. \begin{cases} x = \frac{2t+t^2}{1+t^3}, \\ y = \frac{2t-t^2}{1+t^3}, \quad t_0 = 1. \end{cases}$$

$$16.6. \begin{cases} x = \arcsin \frac{t}{\sqrt{1+t^2}}, \\ y = \arccos \frac{1}{\sqrt{1+t^2}}, \quad t_0 = -1. \end{cases}$$

$$16.7. \begin{cases} x = t(t \operatorname{cost} - 2 \sin t), \\ y = t(t \sin t + 2 \operatorname{cost}), \quad t_0 = \pi/4. \end{cases}$$

$$16.8. \begin{cases} x = \frac{3at}{1+t^2}, \\ y = \frac{3at^2}{1+t^2}, \quad t_0 = 2. \end{cases}$$

$$16.9. \begin{cases} x = 2 \ln(\operatorname{ctg} t) + \operatorname{ctg} t, \\ y = \operatorname{tg} t + \operatorname{ctg} t, \quad t_0 = \pi/4. \end{cases}$$

$$16.10. \begin{cases} x = \frac{1}{2} t^2 - \frac{1}{4} t^4, \\ y = \frac{1}{2} t^2 + \frac{1}{3} t^3, \quad t_0 = 0. \end{cases}$$

$$16.11. \begin{cases} x = at \cos t, \\ y = at \sin t, \quad t_0 = \pi/2. \end{cases}$$

$$16.12. \begin{cases} x = \sin t, \\ y = \cos t, \quad t_0 = \pi/6. \end{cases}$$

$$16.13. \begin{cases} x = \arcsin \frac{t}{\sqrt{1+t^2}}, \\ y = \arccos \frac{1}{\sqrt{1+t^2}}, \quad t_0 = 1. \end{cases}$$

$$16.14. \begin{cases} x = \frac{1 + \ln t}{t^2}, \\ y = \frac{3 + 2 \ln t}{t}, \quad t_0 = 1. \end{cases}$$

$$16.15. \begin{cases} x = \frac{1+t}{t^2}, \\ y = \frac{3}{2t^2} + \frac{2}{t}, \quad t_0 = 2. \end{cases}$$

$$16.16. \begin{cases} x = a \sin^3 t, \\ y = a \cos^3 t, \quad t_0 = \pi/6. \end{cases}$$

$$16.17. \begin{cases} x = a(t \sin t + \cos t), \\ y = a(\sin t - t \cos t), \quad t_0 = \pi/4. \end{cases}$$

$$16.18. \begin{cases} x = \frac{t+1}{t}, \\ y = \frac{t-1}{t}, \quad t_0 = -1. \end{cases}$$

$$16.19. \begin{cases} x = 1 - t^2, \\ y = t - t^3, \quad t_0 = 2. \end{cases}$$

$$16.20. \begin{cases} x = \ln(1+t^2), \\ y = t - \operatorname{arctg} t, \quad t_0 = 1. \end{cases}$$

$$16.21. \begin{cases} x = t(1 - \sin t), \\ y = t \cos t, \quad t_0 = 0. \end{cases}$$

$$16.22. \begin{cases} x = \frac{1+t^3}{t^2-1}, \\ y = \frac{t}{t^2-1}, \quad t_0 = 2. \end{cases}$$

$$16.23. \begin{cases} x = 3 \cos t, \\ y = 4 \sin t, \quad t_0 = \pi/4. \end{cases}$$

$$16.24. \begin{cases} x = t - t^4, \\ y = t^2 - t^3, \quad t_0 = 1. \end{cases}$$

$$16.25. \begin{cases} x = t^3 + 1, \\ y = t^2 + t + 1, \quad t_0 = 1. \end{cases}$$

$$16.26. \begin{cases} x = 2 \cos t, \\ y = \sin t, \quad t_0 = -\pi/3. \end{cases}$$

$$16.27. \begin{cases} x = 2 \operatorname{tg} t, \\ y = 2 \sin^2 t + \sin 2t, \quad t_0 = \pi/4. \end{cases}$$

$$16.28. \begin{cases} x = t^3 + 1, \\ y = t^2, \quad t_0 = -2. \end{cases}$$

$$16.29. \begin{cases} x = \sin t, \\ y = a^t, \quad t_0 = 0. \end{cases}$$

$$16.30. \begin{cases} x = \sin t, \\ y = \cos 2t, \quad t_0 = \pi/6. \end{cases}$$

$$16.31. \begin{cases} x = 2e^t, \\ y = e^{-t}, \quad t_0 = 0. \end{cases}$$

**Задача 17.** Найти производную  $n$ -го порядка.

$$17.1. y = xe^{ax}.$$

$$17.2. y = \sin 2x + \cos(x+1).$$

$$17.3. y = \sqrt[5]{e^{7x-1}}.$$

$$17.4. y = \frac{4x+7}{2x+3}.$$

$$17.5. y = \lg(5x+2).$$

$$17.6. y = a^{3x}.$$

$$17.7. y = \frac{x}{2(3x+2)}.$$

$$17.8. y = \lg(x+4).$$

$$17.9. y = \sqrt{x}.$$

$$17.10. y = \frac{2x+5}{13(3x+1)}.$$

$$17.11. y = 2^{3x+5}.$$

$$17.12. y = \sin(x+1) + \cos 2x.$$

$$17.13. y = \sqrt[3]{e^{2x+1}}.$$

$$17.14. y = \frac{4+15x}{5x+1}.$$

$$17.15. y = \lg(3x+1).$$

$$17.16. y = 7^{5x}.$$

$$17.17. y = \frac{x}{9(4x+9)}.$$

$$17.18. y = \lg(1+x).$$

$$17.19. y = \frac{4}{x}.$$

$$17.20. y = \frac{5x+1}{13(2x+3)}.$$

$$17.21. y = a^{2x+3}.$$

$$17.22. y = \sin(3x+1) + \cos 5x.$$

$$17.23. y = \sqrt{e^{3x+1}}.$$

$$17.24. y = \frac{11+12x}{6x+5}.$$

17.25.  $y = \lg(2x + 7)$ .

17.26.  $y = 2^{kx}$ .

17.27.  $y = \frac{x}{x+1}$ .

17.28.  $y = \log_3(x + 5)$ .

17.29.  $y = \frac{1+x}{1-x}$ .

17.30.  $y = \frac{7x+1}{17(4x+3)}$ .

17.31.  $y = 3^{2x+5}$ .

**Задача 18.** Найти производную указанного порядка.

18.1.  $y = (2x^2 - 7)\ln(x - 1)$ ,  $y^V = ?$

18.2.  $y = (3 - x^2)\ln^2 x$ ,  $y^{III} = ?$

18.3.  $y = x \cos x^2$ ,  $y^{III} = ?$

18.4.  $y = \frac{\ln(x-1)}{\sqrt{x-1}}$ ,  $y^{III} = ?$

18.5.  $y = \frac{\log_2 x}{x^3}$ ,  $y^{III} = ?$

18.6.  $y = (4x^3 + 5)e^{2x+1}$ ,  $y^V = ?$

18.7.  $y = x^2 \sin(5x - 3)$ ,  $y^{III} = ?$

18.8.  $y = \frac{\ln x}{x^2}$ ,  $y^{IV} = ?$

18.9.  $y = (2x + 3)\ln^2 x$ ,  $y^{III} = ?$

18.10.  $y = (1 + x^2)\operatorname{arctg} x$ ,  $y^{III} = ?$

18.11.  $y = \frac{\ln x}{x^3}$ ,  $y^{IV} = ?$

18.12.  $y = (4x + 3) \cdot 2^{-x}$ ,  $y^V = ?$

18.13.  $y = e^{1-2x} \cdot \sin(2+3x)$ ,  $y^{IV} = ?$

18.14.  $y = \frac{\ln(3+x)}{3+x}$ ,  $y^{III} = ?$

18.15.  $y = (2x^3 + 1)\cos x$ ,  $y^V = ?$

18.16.  $y = (x^2 + 3)\ln(x - 3)$ ,  $y^{IV} = ?$

18.17.  $y = (1 - x - x^2)e^{(x-1)/2}$ ,  $y^{IV} = ?$

18.18.  $y = \frac{1}{x}\sin 2x$ ,  $y^{III} = ?$

18.19.  $y = (x + 7)\ln(x + 4)$ ,  $y^V = ?$

18.20.  $y = (3x - 7) \cdot 3^{-x}$ ,  $y^{IV} = ?$

18.21.  $y = \frac{\ln(2x+5)}{2x+5}$ ,  $y^{III} = ?$

18.22.  $y = e^{x/2} \cdot \sin 2x$ ,  $y^{IV} = ?$

18.23.  $y = \frac{\ln x}{x^5}$ ,  $y^{III} = ?$

18.24.  $y = x \ln(1 - 3x)$ ,  $y^{IV} = ?$

$$18.25. y = (x^2 + 3x + 1)e^{3x+2}, \quad y' = ?$$

$$18.26. y = (5x - 8) \cdot 2^{-x}, \quad y'' = ?$$

$$18.27. y = \frac{\ln(x-2)}{x-2}, \quad y' = ?$$

18.28.

$$y = e^{-x} \cdot (\cos 2x - 3 \sin 2x), \quad y'' = ?$$

$$18.29. y = (5x - 1) \ln^2 x, \quad y''' = ?$$

$$18.30. y = \frac{\log_3 x}{x^2}, \quad y'' = ?$$

$$18.31. y = (x^3 + 3)e^{4x+3}, \quad y'' = ?$$

**Задача 19.** Найти производную второго порядка  $y''_{xx}$  от функции, заданной параметрически.

$$19.1. \begin{cases} x = \cos 2t, \\ y = 2 \sec^2 t. \end{cases}$$

$$19.2. \begin{cases} x = \sqrt{1-t^2}, \\ y = 1/t. \end{cases}$$

$$19.3. \begin{cases} x = e^t \cos t, \\ y = e^t \sin t. \end{cases}$$

$$19.4. \begin{cases} x = \operatorname{sh}^2 t, \\ y = 1/\operatorname{ch}^2 t. \end{cases}$$

$$19.5. \begin{cases} x = t + \sin t, \\ y = 2 - \cos t. \end{cases}$$

$$19.6. \begin{cases} x = 1/t, \\ y = 1/(1+t^2). \end{cases}$$

$$19.7. \begin{cases} x = \sqrt{t}, \\ y = 1/\sqrt{1-t}. \end{cases}$$

$$19.8. \begin{cases} x = \sin t, \\ y = \sec t. \end{cases}$$

$$19.9. \begin{cases} x = \operatorname{tg} t, \\ y = 1/\sin 2t. \end{cases}$$

$$19.10. \begin{cases} x = \sqrt{t-1}, \\ y = t/\sqrt{1-t}. \end{cases}$$

$$19.11. \begin{cases} x = \sqrt{t}, \\ y = \sqrt[3]{t-1}. \end{cases}$$

$$19.12. \begin{cases} x = \cos t/(1+2 \cos t), \\ y = \sin t/(1+2 \cos t). \end{cases}$$

$$19.13. \begin{cases} x = \sqrt{t^3-1}, \\ y = \ln t. \end{cases}$$

$$19.14. \begin{cases} x = \operatorname{sh} t, \\ y = \operatorname{th}^2 t. \end{cases}$$

$$19.15. \begin{cases} x = \sqrt{t-1}, \\ y = 1/\sqrt{t}. \end{cases}$$

$$19.16. \begin{cases} x = \cos^2 t, \\ y = \operatorname{tg}^2 t. \end{cases}$$

$$19.17. \begin{cases} x = \sqrt{t-3}, \\ y = \ln(t-2). \end{cases}$$

$$19.18. \begin{cases} x = \sin t, \\ y = \ln \cos t. \end{cases}$$

$$19.19. \begin{cases} x = t + \sin t, \\ y = 2 + \cos t. \end{cases}$$

$$19.20. \begin{cases} x = t - \sin t, \\ y = 2 - \cos t. \end{cases}$$

$$19.21. \begin{cases} x = \cos t, \\ y = \ln \sin t. \end{cases}$$

$$19.22. \begin{cases} x = \cos t + t \sin t, \\ y = \sin t - t \cos t. \end{cases}$$

$$19.23. \begin{cases} x = e^t, \\ y = \arcsin t. \end{cases}$$

$$19.24. \begin{cases} x = \cos t, \\ y = \sin^4(t/2). \end{cases}$$

$$19.25. \begin{cases} x = \operatorname{ch} t, \\ y = \sqrt[3]{\operatorname{sh}^2 t}. \end{cases}$$

$$19.26. \begin{cases} x = \operatorname{arctg} t, \\ y = t^2/2. \end{cases}$$

$$19.27. \begin{cases} x = 2(t - \sin t), \\ y = 4(2 + \cos t). \end{cases}$$

$$19.28. \begin{cases} x = \sin t - t \cos t, \\ y = \cos t + t \sin t. \end{cases}$$

$$19.29. \begin{cases} x = 1/t^2, \\ y = 1/(t^2 + 1). \end{cases}$$

$$19.30. \begin{cases} x = \cos t + \sin t, \\ y = \sin 2t. \end{cases}$$

$$19.31. \begin{cases} x = \ln t, \\ y = \operatorname{arctg} t. \end{cases}$$

**Задача 20.** Показать, что функция  $y$  удовлетворяет уравнению (1).

$$20.1. \begin{cases} y = x e^{-x^2/2}, \\ xy' = (1 - x^2)y. \end{cases} \quad (1)$$

$$20.2. \begin{cases} y = \frac{\sin x}{x}, \\ xy' + y = \cos x. \end{cases} \quad (1)$$

$$20.3. \begin{cases} y = 5e^{-2x} + e^x/3, \\ y' + 2y = e^x. \end{cases} \quad (1)$$

$$20.4. \begin{cases} y = 2 + c\sqrt{1-x^2}, \\ (1-x^2)y' + xy = 2x. \end{cases} \quad (1)$$

$$20.5. \quad y = x\sqrt{1-x^2}, \\ yy' = x - 2x^3. \quad (1)$$

$$20.7. \quad y = -\frac{1}{3x+c}, \\ y' = 3y^2. \quad (1)$$

$$20.9. \quad y = \sqrt{x^2 - cx}, \\ (x^2 + y^2)dx - 2xydy = 0. \quad (1)$$

$$20.11. \quad y = e^{\operatorname{tg}(x/2)}, \\ y' \sin x = y \ln y. \quad (1)$$

$$20.13. \quad y = \frac{b+x}{1+bx}, \\ y - xy' = b(1+x^2y'). \quad (1)$$

$$20.15. \quad y = \sqrt{\ln\left(\frac{1+e^x}{2}\right)^2 + 1}, \\ (1+e^x)yy' = e^x. \quad (1)$$

$$20.17. \quad y = -\sqrt{\frac{2}{x^2} - 1}, \\ 1 + y^2 + xy' = 0. \quad (1)$$

$$20.19. \quad y = a + \frac{7x}{ax+1}, \\ y - xy' = a(1+x^2y'). \quad (1)$$

$$20.21. \quad y = \sqrt[4]{\sqrt{x} + \sqrt{x+1}}, \\ 8xy' - y = \frac{-1}{y^3\sqrt{x+1}}. \quad (1)$$

$$20.6. \quad y = \frac{c}{\cos x}, \\ y' - \operatorname{tg} x \cdot y = 0. \quad (1)$$

$$20.8. \quad y = \ln(c + e^x), \\ y' = e^{x-y}. \quad (1)$$

$$20.10. \quad y = x(c - \ln x), \\ (x-y)dx + xdy = 0. \quad (1)$$

$$20.12. \quad y = \frac{1+x}{1-x}, \\ y' = \frac{1+y}{1+x^2}. \quad (1)$$

$$20.14. \quad y = \sqrt[3]{2+3x-3x^2}, \\ yy' = \frac{1-2x}{y}. \quad (1)$$

$$20.16. \quad y = \operatorname{tg} \ln 3x, \\ (1+y^2)dx = xdy. \quad (1)$$

$$20.18. \quad y = \sqrt[3]{x - \ln x - 1}, \\ \ln x + y^3 - 3xy^2y' = 0. \quad (1)$$

$$20.20. \quad y = a \operatorname{tg} \sqrt{\frac{a}{x} - 1}, \\ a^2 + y^2 + 2x\sqrt{ax - x^2}y' = 0. \quad (1)$$

$$20.22. \quad y = (x+1)e^{x^2}, \\ y' - 2xy = 2xe^{x^2}. \quad (1)$$



$$20.23. \quad y = \frac{2x}{x^3 + 1} + \frac{1}{x},$$

$$x(x^3 + 1)y' + (2x^3 - 1)y = \frac{x^3 - 2}{x}. \quad (1)$$

$$20.25. \quad y = -x \cos x + 3x,$$

$$xy' = y + x^2 \sin x. \quad (1)$$

$$20.27. \quad y = \frac{x}{x-1} + x^2,$$

$$x(x-1)y' + y = x^2(2x-1). \quad (1)$$

$$20.29. \quad y = (x+1)^n (e^x - 1),$$

$$y' - \frac{ny}{x+1} = e^x(1+x)^n. \quad (1)$$

$$20.31. \quad y = -\sqrt{x^4 - x^2},$$

$$xyy' - y^2 = x^4. \quad (1)$$

$$20.24. \quad y = e^{x+x^2} + 2e^x,$$

$$y' - y = 2xe^{x+x^2}. \quad (1)$$

$$y = 1/\sqrt{\sin x + x},$$

$$20.26. \quad 2 \sin x \cdot y' + y \cos x =$$

$$= y^3(x \cos x - \sin x). \quad (1)$$

$$20.28. \quad y = \frac{x}{\cos x},$$

$$y' - y \operatorname{tg} x = \sec x. \quad (1)$$

$$y = 2 \frac{\sin x}{x} + \cos x,$$

$$20.30. \quad x \sin x \cdot y' + (\sin x - x \cos x)y =$$

$$= \sin x \cdot \cos x - x. \quad (1)$$

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