

VII. КРАТНЫЕ ИНТЕГРАЛЫ

Теоретические вопросы

1. Определение двойного и тройного интегралов. Их геометрический и физический смысл.
2. Основные свойства двойных и тройных интегралов.
3. Теорема о среднем для двойного и тройного интегралов.
4. Вычисление двойных интегралов двумя последовательными интегрированиями (случай прямоугольной области).
5. Вычисление двойных интегралов двумя последовательными интегрированиями (общий случай).
6. Замена переменных в двойном интеграле.
7. Якобиан, его геометрический смысл.
8. Двойной интеграл в полярных координатах.
9. Тройной интеграл в цилиндрических координатах.
10. Тройной интеграл в сферических координатах.

Теоретические упражнения

1. Пользуясь определением двойного интеграла, доказать, что

$$\iint_{x^2+y^2 \leq R^2} x^m y^n dx dy = 0,$$

если m и n - натуральные числа, и, по меньшей мере, одно из них нечетно.

2. С помощью теоремы о среднем найти

$$\lim_{R \rightarrow 0} \frac{1}{\pi R^2} \iint_{x^2+y^2 \leq R^2} f(x, y) dx dy,$$

где $f(x, y)$ - непрерывная функция.

3. Оценить интеграл

$$\iiint_{x^2+y^2+z^2 \leq R^2} \frac{dx dy dz}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}, \quad x_0^2 + y_0^2 + z_0^2 > R^2,$$

т.е. указать, между какими значениями заключена его величина.

4. Вычислить двойной интеграл

$$\iint_D f(x, y) dx dy,$$

если область D - прямоугольник $\{a \leq x \leq b, c \leq y \leq d\}$, а $f(x, y) = F''_{xy}(x, y)$.

5. Доказать равенство

$$\iint_D f(x)g(y) dx dy = \int_a^b f(x) dx \int_c^d g(y) dy$$

если область D - прямоугольник $\{a \leq x \leq b, c \leq y \leq d\}$.

6. Доказать формулу Дирихле

$$\int_0^a dx \int_0^x f(x, y) dy = \int_0^a dy \int_y^a f(x, y) dx, \quad a > 0.$$

7. Пользуясь формулой Дирихле, доказать равенство

$$\int_0^a dy \int_0^y f(x) dx = \int_0^a (a-x) f(x) dx.$$

8. Какой из интегралов больше

$$\int_0^1 dx \int_0^1 dy \int_0^1 f(x, y, z) dz \quad \text{или} \quad \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} f(x, y, z) dz,$$

если $f(x, y, z) > 0$?

Расчетные задания

Задача 1. Изменить порядок интегрирования.

$$1.1. \int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^0 f dx + \int_{-1}^0 dy \int_{-\sqrt{-y}}^0 f dx.$$

$$1.2. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{-y}}^0 f dx.$$

$$1.3. \int_0^1 dy \int_0^y f dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f dx.$$

$$1.4. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^2 dy \int_0^{\sqrt{2-y}} f dx.$$

$$1.5. \int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^0 f dy + \int_{-1}^0 dx \int_x^0 f dy.$$

$$1.6. \int_0^{1/\sqrt{2}} dy \int_0^{\arcsin y} f dx + \int_{1/\sqrt{2}}^1 dy \int_0^{\arccos y} f dx.$$

$$1.7. \int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f dx.$$

$$1.8. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^e dy \int_{-1}^{-\ln y} f dx.$$

$$1.9. \int_{-\sqrt{2}}^{-1} dx \int_0^{\sqrt{2-x^2}} f dy + \int_{-1}^0 dx \int_0^{x^2} f dy.$$

$$1.10. \int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^0 f dy + \int_{-\sqrt{3}}^0 dx \int_{\sqrt{4-x^2}-2}^0 f dy.$$

$$1.11. \int_0^1 dx \int_{1-x^2}^1 f dy + \int_1^e dx \int_{\ln x}^1 f dy.$$

$$1.12. \int_0^1 dy \int_0^{\sqrt[3]{y}} f dx + \int_1^2 dy \int_0^{2-y} f dx.$$

$$1.13. \int_0^{\pi/4} dy \int_0^{\sin y} f dx + \int_{\pi/4}^{\pi/2} dy \int_0^{\cos y} f dx.$$

$$1.14. \int_{-2}^{-1} dx \int_{-(2+x)}^0 f dy + \int_{-1}^0 dx \int_{\sqrt[3]{x}}^0 f dy.$$

$$1.15. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^e dy \int_{\ln y}^1 f dx.$$

$$1.16. \int_0^1 dy \int_{-\sqrt{y}}^0 f dx + \int_1^2 dy \int_{-\sqrt{2-y}}^0 f dx.$$

$$1.17. \int_0^1 dy \int_{-y}^0 f dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f dx.$$

$$1.18. \int_0^1 dy \int_0^{y^2} f dx + \int_1^2 dy \int_0^{2-y} f dx.$$

$$1.19. \int_0^{\sqrt{3}} dx \int_{\sqrt{4-x^2}-2}^0 f dy + \int_{\sqrt{3}}^2 dx \int_{-\sqrt{4-x^2}}^0 f dy.$$

$$1.20. \int_{-2}^{-1} dy \int_{-(2+y)}^0 f dx + \int_{-1}^0 dy \int_{\sqrt[3]{y}}^0 f dx.$$

$$1.21. \int_0^1 dy \int_0^y f dx + \int_1^e dy \int_{\ln y}^1 f dx.$$

$$1.22. \int_0^1 dx \int_0^{x^2} f dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f dy.$$

$$1.23. \int_0^{\pi/4} dx \int_0^{\sin x} f dy + \int_{\pi/4}^{\pi/2} dx \int_0^{\cos x} f dy.$$

$$1.24. \int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^0 f dx + \int_{-1}^0 dy \int_{\sqrt{y}}^0 f dx.$$

$$1.25. \int_0^1 dx \int_0^{x^2} f dy + \int_1^2 dx \int_0^{2-x} f dy.$$

$$1.26. \int_0^{\sqrt{3}} dx \int_0^{2-\sqrt{4-x^2}} f dy + \int_{\sqrt{3}}^2 dx \int_0^{\sqrt{4-x^2}} f dy.$$

$$1.27. \int_0^1 dx \int_{-\sqrt{x}}^0 f dy + \int_1^2 dx \int_{-\sqrt{2-x}}^0 f dy.$$

$$1.28. \int_0^1 dx \int_0^x f dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f dy.$$

$$1.29. \int_0^1 dy \int_0^{\sqrt{y}} f dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f dx.$$

$$1.30. \int_0^1 dx \int_0^{\sqrt{x}} f dy + \int_1^2 dx \int_0^{\sqrt{2-x}} f dy.$$

$$1.31. \int_{-2}^{-\sqrt{3}} dx \int_0^{\sqrt{4-x^2}} f dy + \int_{-\sqrt{3}}^0 dx \int_0^{2-\sqrt{4-x^2}} f dy.$$

Задача 2. Вычислить.

$$2.1. \iint_D (12x^2y^2 + 16x^3y^3) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt{x}.$$

$$2.2. \iint_D (9x^2y^2 + 48x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt{x}, y=-x^2.$$

$$2.3. \iint_D (36x^2y^2 - 96x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^3.$$

$$2.4. \iint_D (18x^2y^2 + 32x^3y^3) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt[3]{x}.$$

$$2.5. \iint_D (27x^2y^2 + 48x^3y^3) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt[3]{x}.$$

$$2.7. \iint_D (18x^2y^2 + 32x^3y^3) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt{x}.$$

$$2.9. \iint_D (4xy + 3x^2y^2) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt{x}.$$

$$2.11. \iint_D (8xy + 9x^2y^2) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^3.$$

$$2.13. \iint_D (12xy + 27x^2y^2) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt[3]{x}.$$

$$2.15. \iint_D \left(\frac{4}{5}xy + \frac{9}{11}x^2y^2 \right) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt{x}.$$

$$2.17. \iint_D (24xy - 48x^2y^3) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt{x}.$$

$$2.19. \iint_D (4xy + 16x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^3.$$

$$2.21. \iint_D (44xy + 16x^3y^3) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt[3]{x}.$$

$$2.6. \iint_D (18x^2y^2 + 32x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^2.$$

$$2.8. \iint_D (27x^2y^2 + 48x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt{x}, y=-x^3.$$

$$2.10. \iint_D (12xy + 9x^2y^2) dx dy;$$

$$D: x=1, y=\sqrt{x}, y=-x^2.$$

$$2.12. \iint_D (24xy + 18x^2y^2) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt[3]{x}.$$

$$2.14. \iint_D (8xy + 18x^2y^2) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^2.$$

$$2.16. \iint_D \left(\frac{4}{5}xy + 9x^2y^2 \right) dx dy;$$

$$D: x=1, y=\sqrt{x}, y=-x^3.$$

$$2.18. \iint_D (6xy + 24x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt{x}, y=-x^2.$$

$$2.20. \iint_D (4xy + 16x^3y^3) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt[3]{x}.$$

$$2.22. \iint_D (4xy + 176x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^3.$$

$$2.23. \iint_D (xy - 4x^3y^3) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt{x}.$$

$$2.25. \iint_D \left(6x^2y^2 + \frac{25}{3}x^4y^4 \right) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt{x}.$$

$$2.27. \iint_D \left(3x^2y^2 + \frac{50}{3}x^4y^4 \right) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^3.$$

$$2.29. \iint_D (54x^2y^2 + 150x^4y^4) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt[3]{x}.$$

$$2.31. \iint_D (54x^2y^2 + 150x^4y^4) dx dy;$$

$$D: x=1, y=x^3, y=-\sqrt{x}.$$

$$2.24. \iint_D (4xy + 176x^3y^3) dx dy;$$

$$D: x=1, y=\sqrt{x}, y=-x^3.$$

$$2.26. \iint_D (9x^2y^2 + 25x^4y^4) dx dy;$$

$$D: x=1, y=\sqrt{x}, y=-x^2.$$

$$2.28. \iint_D (9x^2y^2 + 25x^4y^4) dx dy;$$

$$D: x=1, y=x^2, y=-\sqrt[3]{x}.$$

$$2.30. \iint_D (xy + 19x^5y^5) dx dy;$$

$$D: x=1, y=\sqrt[3]{x}, y=-x^2.$$

Задача 3. Вычислить.

$$3.1. \iint_D ye^{xy/2} dx dy;$$

$$D: y=\ln 2, y=\ln 3, x=2, x=4.$$

$$3.3. \iint_D y \cos xy dx dy;$$

$$D: y=\pi/2, y=\pi, x=1, x=2.$$

$$3.5. \iint_D y \sin xy dx dy;$$

$$D: y=\pi/2, y=\pi, x=1, x=2.$$

$$3.2. \iint_D y^2 \sin \frac{xy}{2} dx dy;$$

$$D: x=0, y=\sqrt{\pi}, y=\frac{x}{2}.$$

$$3.4. \iint_D y^2 e^{-xy/4} dx dy;$$

$$D: x=0, y=2, y=x.$$

$$3.6. \iint_D y^2 \cos \frac{xy}{2} dx dy;$$

$$D: x=0, y=\sqrt{\pi/2}, y=x/2.$$

$$3.7. \iint_D 4ye^{2xy} dx dy;$$

$$D: y = \ln 3, y = \ln 4, x = \frac{1}{2}, x = 1.$$

$$3.9. \iint_D y \cos 2xy dx dy;$$

$$D: y = \frac{\pi}{2}, y = \pi, x = \frac{1}{2}, x = 1.$$

$$3.11. \iint_D 12y \sin 2xy dx dy;$$

$$D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 2, x = 3.$$

$$3.13. \iint_D ye^{xy/4} dx dy;$$

$$D: y = \ln 2, y = \ln 3, x = 4, x = 8.$$

$$3.15. \iint_D 2y \cos 2xy dx dy;$$

$$D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 1, x = 2.$$

$$3.17. \iint_D y \sin xy dx dy;$$

$$D: y = \pi, y = 2\pi, x = \frac{1}{2}, x = 1.$$

$$3.19. \iint_D 8ye^{4xy} dx dy;$$

$$D: y = \ln 3, y = \ln 4, x = \frac{1}{4}, x = \frac{1}{2}.$$

$$3.8. \iint_D 4y^2 \sin xy dx dy;$$

$$D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = x.$$

$$3.10. \iint_D y^2 e^{-xy/8} dx dy;$$

$$D: x = 0, y = 2, y = \frac{x}{2}.$$

$$3.12. \iint_D y^2 \cos xy dx dy;$$

$$D: x = 0, y = \sqrt{\pi}, y = x.$$

$$3.14. \iint_D y^2 \sin 2xy dx dy;$$

$$D: x = 0, y = \sqrt{2\pi}, y = 2x.$$

$$3.16. \iint_D y^2 e^{-xy/2} dx dy;$$

$$D: x = 0, y = \sqrt{2}, y = x.$$

$$3.18. \iint_D y^2 \cos 2xy dx dy;$$

$$D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}.$$

$$3.20. \iint_D 3y^2 \sin \frac{xy}{2} dx dy;$$

$$D: x = 0, y = \sqrt{\frac{4\pi}{3}}, y = \frac{2}{3}x.$$

$$3.21. \iint_D y \cos xy \, dx dy;$$

$$D: y = \pi, y = 3\pi, x = 1/2, x = 1.$$

$$3.23. \iint_D y \sin 2xy \, dx dy;$$

$$D: y = \pi/2, y = 3\pi/2, x = 1/2, x = 2.$$

$$3.25. \iint_D 6ye^{xy/3} \, dx dy;$$

$$D: y = \ln 2, y = \ln 3, x = 3, x = 6.$$

$$3.27. \iint_D y \cos 2xy \, dx dy;$$

$$D: y = \pi/2, y = 3\pi/2, x = 1/2, x = 2.$$

$$3.29. \iint_D 3y \sin xy \, dx dy;$$

$$D: y = \pi/2, y = 3\pi, x = 1, x = 3.$$

$$3.31. \iint_D 12ye^{6xy} \, dx dy;$$

$$D: y = \ln 3, y = \ln 4, x = 1/6, x = 1/3.$$

$$3.22. \iint_D y^2 e^{-xy/2} \, dx dy;$$

$$D: x = 0, y = 1, y = \frac{x}{2}.$$

$$3.24. \iint_D y^2 \cos xy \, dx dy;$$

$$D: x = 0, y = \sqrt{\pi}, y = 2x.$$

$$3.26. \iint_D y^2 \sin \frac{xy}{2} \, dx dy;$$

$$D: x = 0, y = \sqrt{\pi}, y = x.$$

$$3.28. \iint_D y^2 e^{-xy/8} \, dx dy;$$

$$D: x = 0, y = 4, y = 2x.$$

$$3.30. \iint_D y^2 \cos \frac{xy}{2} \, dx dy;$$

$$D: x = 0, y = \sqrt{2\pi}, y = 2x.$$

Задача 4. Вычислить.

$$4.1. \iiint_V 2e^{xy} \, dx \, dy \, dz;$$

$$V \begin{cases} x = 0, y = 1, y = x, \\ z = 0, z = 1. \end{cases}$$

$$4.2. \iiint_V x^2 z \sin(xyz) \, dx \, dy \, dz;$$

$$V \begin{cases} x = 2, y = \pi, z = 1, \\ x = 0, y = 1, z = 0. \end{cases}$$

$$4.3. \iiint_V y^2 \operatorname{ch}(2xy) \, dx \, dy \, dz;$$

$$V \begin{cases} x=0, y=-2, y=4x, \\ z=0, z=2. \end{cases}$$

$$4.5. \iiint_V x^2 \operatorname{sh}(3xy) \, dx \, dy \, dz;$$

$$V \begin{cases} x=1, y=2x, y=0, \\ z=0, z=36. \end{cases}$$

$$4.7. \iiint_V y^2 \cos\left(\frac{\pi}{4}xy\right) \, dx \, dy \, dz;$$

$$V \begin{cases} x=0, y=-1, y=x/2, \\ z=0, z=-\pi^2. \end{cases}$$

$$4.9. \iiint_V y^2 e^{-xy} \, dx \, dy \, dz;$$

$$V \begin{cases} x=0, y=-2, y=4x, \\ z=0, z=1. \end{cases}$$

$$4.11. \iiint_V y^2 \operatorname{ch}(2xy) \, dx \, dy \, dz;$$

$$V \begin{cases} x=0, y=1, y=x, \\ z=0, z=8. \end{cases}$$

$$4.13. \iiint_V y^2 e^{xy/2} \, dx \, dy \, dz;$$

$$V \begin{cases} x=0, y=2, y=2x, \\ z=0, z=-1. \end{cases}$$

$$4.15. \iiint_V y^2 \cos\left(\frac{\pi xy}{2}\right) \, dx \, dy \, dz;$$

$$V \begin{cases} x=0, y=-1, y=x, \\ z=0, z=2\pi^2. \end{cases}$$

$$4.4. \iiint_V 8y^2 z e^{2xyz} \, dx \, dy \, dz;$$

$$V \begin{cases} x=-1, y=2, z=1, \\ x=0, y=0, z=0. \end{cases}$$

$$4.6. \iiint_V y^2 z \cos(xyz) \, dx \, dy \, dz;$$

$$V \begin{cases} x=1, y=2\pi, z=2, \\ x=0, y=1, z=0. \end{cases}$$

$$4.8. \iiint_V x^2 z \sin\frac{xyz}{4} \, dx \, dy \, dz;$$

$$V \begin{cases} x=1, y=2\pi, z=4, \\ x=0, y=0, z=0. \end{cases}$$

$$4.10. \iiint_V 2y^2 z e^{2xyz} \, dx \, dy \, dz;$$

$$V \begin{cases} x=1, y=1, z=1, \\ x=0, y=0, z=0. \end{cases}$$

$$4.12. \iiint_V x^2 z \operatorname{sh}(xyz) \, dx \, dy \, dz;$$

$$V \begin{cases} x=2, y=1, z=1, \\ x=0, y=0, z=0. \end{cases}$$

$$4.14. \iiint_V y^2 z \cos\frac{xyz}{3} \, dx \, dy \, dz;$$

$$V \begin{cases} x=3, y=1, z=2\pi, \\ x=0, y=0, z=0. \end{cases}$$

$$4.16. \iiint_V x^2 z \operatorname{sh}(xyz) \, dx \, dy \, dz;$$

$$V \begin{cases} x=1, y=-1, z=1, \\ x=0, y=0, z=0. \end{cases}$$

$$\iiint_V y^2 \cos(\pi xy) dx dy dz;$$

4.17.
$$V \begin{cases} x = 0, y = 1, y = 2x, \\ z = 0, z = \pi^2. \end{cases}$$

$$\iiint_V x^2 \operatorname{sh}(2xy) dx dy dz;$$

4.19.
$$V \begin{cases} x = -1, y = x, y = 0, \\ z = 0, z = 8. \end{cases}$$

$$\iiint_V y^2 \operatorname{ch}(xy) dx dy dz;$$

4.21.
$$V \begin{cases} x = 0, y = -1, y = x, \\ z = 0, z = 2. \end{cases}$$

$$\iiint_V x^2 \cos\left(\frac{\pi}{2} xy\right) dx dy dz;$$

4.23.
$$V \begin{cases} x = 2, y = x, y = 0, \\ z = 0, z = \pi. \end{cases}$$

$$\iiint_V x^2 \cos(\pi xy) dx dy dz;$$

4.25.
$$V \begin{cases} x = 1, y = 2x, y = 0, \\ z = 0, z = 4\pi. \end{cases}$$

$$\iiint_V y^2 \operatorname{ch}(3xy) dx dy dz;$$

4.27.
$$V \begin{cases} x = 0, y = 2, y = 6x, \\ z = 0, z = -3. \end{cases}$$

$$\iiint_V 2x^2 z \operatorname{sh}(2xyz) dx dy dz;$$

4.18.
$$V \begin{cases} x = 2, y = 1/2, z = 1/2, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$\iiint_V x^2 z \sin\frac{xyz}{2} dx dy dz;$$

4.20.
$$V \begin{cases} x = 1, y = 4, z = \pi, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$\iiint_V x^2 z \operatorname{ch}(xyz) dx dy dz;$$

4.22.
$$V \begin{cases} x = 1, y = 1, z = 1, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$\iiint_V y^2 z \cos\frac{xyz}{9} dx dy dz;$$

4.24.
$$V \begin{cases} x = 9, y = 1, z = 2\pi, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$\iiint_V y^2 z \operatorname{ch}\left(\frac{xyz}{2}\right) dx dy dz;$$

4.26.
$$V \begin{cases} x = 2, y = -1, z = 2, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$\iiint_V 2y^2 z \operatorname{ch}(2xyz) dx dy dz;$$

4.28.
$$V \begin{cases} x = \frac{1}{2}, y = 2, z = -1, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$\iiint_V x^2 \sin(4\pi xy) \, dx \, dy \, dz;$$

4.29.
$$V \begin{cases} x=1, y=x/2, y=0, \\ z=0, z=8\pi. \end{cases}$$

$$\iiint_V 8y^2 z e^{-xyz} \, dx \, dy \, dz;$$

4.30.
$$V \begin{cases} x=2, y=-1, z=2, \\ x=0, y=0, z=0. \end{cases}$$

$$\iiint_V x^2 \operatorname{sh}(xy) \, dx \, dy \, dz;$$

4.31.
$$V \begin{cases} x=2, y=x/2, y=0, \\ z=0, z=1. \end{cases}$$

Задача 5. Вычислить.

$$\iiint_V x \, dx \, dy \, dz;$$

5.1.
$$V: y=10x, y=0, x=1, \\ z=xy, z=0.$$

$$\iiint_V \frac{dx \, dy \, dz}{\left(1 + \frac{x}{3} + \frac{y}{4} + \frac{z}{8}\right)^4};$$

5.2.
$$V: 1 + \frac{x}{3} + \frac{y}{4} + \frac{z}{8} = 1, \\ x=0, y=0, z=0.$$

$$\iiint_V 15(y^2 + z^2) \, dx \, dy \, dz;$$

5.3.
$$V: z=x+y, x+y \leq 1, \\ x=0, y=0, z \leq 0.$$

$$\iiint_V (3x+4y) \, dx \, dy \, dz;$$

5.4.
$$V: y=x, y=0, x=1, \\ z=5(x^2+y^2), z=0.$$

$$\iiint_V (1+2x^3) \, dx \, dy \, dz;$$

5.5.
$$V: y=9x, y=0, x=1, \\ z=\sqrt{xy}, z=0.$$

$$\iiint_V (27+54y^3) \, dx \, dy \, dz;$$

5.6.
$$V: y=x, y=0, x=1, \\ z=\sqrt{xy}, z=0.$$

$$\iiint_V y \, dx \, dy \, dz;$$

5.7. $V : y = 15x, y = 0, x = 1,$
 $z = xy, z = 0.$

$$\iiint_V (3x^2 + y^2) \, dx \, dy \, dz;$$

5.9. $V : z = 10y, x + y = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V (4 + 8z^3) \, dx \, dy \, dz;$$

5.11. $V : y = x, y = 0, x = 1,$
 $z = \sqrt{xy}, z = 0.$

$$\iiint_V 21xz \, dx \, dy \, dz;$$

5.13. $V : y = x, y = 0, x = 2,$
 $z = xy, z = 0.$

$$\iiint_V (x^2 + 3y^2) \, dx \, dy \, dz;$$

5.15. $V : z = 10x, x + y = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V \left(\frac{10}{3}x + \frac{5}{3} \right) \, dx \, dy \, dz;$$

5.17. $V : y = 9x, y = 0, x = 1,$
 $z = \sqrt{xy}, z = 0.$

$$\iiint_V \frac{dx \, dy \, dz}{\left(1 + \frac{x}{16} + \frac{y}{8} + \frac{z}{3} \right)^5};$$

5.8. $V : \frac{x}{16} + \frac{y}{5} + \frac{z}{3} = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V (15x + 30z) \, dx \, dy \, dz;$$

5.10. $V : z = x^2 + 3y^2, z = 0,$
 $y = x, y = 0, z = 0.$

$$\iiint_V (1 + 2x) \, dx \, dy \, dz;$$

5.12. $V : y = 36x, y = 0, x = 1,$
 $z = \sqrt{xy}, z = 0.$

$$\iiint_V \frac{dx \, dy \, dz}{\left(1 + \frac{x}{10} + \frac{y}{8} + \frac{z}{3} \right)^6};$$

5.14. $V : \frac{x}{10} + \frac{y}{8} + \frac{z}{3} = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V (60y + 90z) \, dx \, dy \, dz;$$

5.16. $V : y = x, y = 0, x = 1,$
 $z = x^2 + y^2, z = 0.$

$$\iiint_V (9 + 18z) \, dx \, dy \, dz;$$

5.18. $V : y = 4x, y = 0, x = 1,$
 $z = \sqrt{xy}, z = 0.$

$$\iiint_V 3y^2 \, dx \, dy \, dz;$$

5.19. $V : y = 2x, y = 0, x = 2,$
 $z = xy, z = 0.$

$$\iiint_V x^2 \, dx \, dy \, dz;$$

5.21. $V : z = 10(x + 3y), x + y = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V 63(1 + 2\sqrt{y}) \, dx \, dy \, dz;$$

5.23. $V : y = x, y = 0, x = 1,$
 $z = \sqrt{xy}, z = 0.$

$$\iiint_V \frac{dx \, dy \, dz}{\left(1 + \frac{x}{6} + \frac{y}{4} + \frac{z}{16}\right)^6};$$

5.25. $V : \frac{x}{6} + \frac{y}{4} + \frac{z}{16} = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V y^2 \, dx \, dy \, dz;$$

5.27. $V : z = 10(3x + y), x + y = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V \frac{dx \, dy \, dz}{\left(1 + \frac{x}{2} + \frac{y}{4} + \frac{z}{6}\right)^6};$$

5.20. $V : \frac{x}{2} + \frac{y}{4} + \frac{z}{6} = 1,$
 $x = 0, y = 0, z = 0.$

$$\iiint_V (8y + 12z) \, dx \, dy \, dz;$$

5.22. $V : y = x, y = 0, x = 1,$
 $z = 3x^2 + 2y^2, z = 0.$

$$\iiint_V (x + y) \, dx \, dy \, dz;$$

5.24. $V : y = x, y = 0, x = 1,$
 $z = 30x^2 + 60y^2, z = 0.$

$$\iiint_V xyz \, dx \, dy \, dz;$$

5.26. $V : y = x, y = 0, x = 2,$
 $z = xy, z = 0.$

$$\iiint_V \left(5x + \frac{3z}{2}\right) \, dx \, dy \, dz;$$

5.28. $V : y = x, y = 0, x = 2,$
 $z = x^2 + 15y^2, z = 0.$

$$\iiint_V (x^2 + 4y^2) dx dy dz;$$

$$5.29. V: z = 20(2x + y), x + y = 1, \\ x = 0, y = 0, z = 0.$$

$$\iiint_V x^2 z dx dy dz;$$

$$5.31. V: y = 3x, y = 0, x = 2, \\ z = xy, z = 0.$$

$$\iiint_V \frac{dx dy dz}{\left(1 + \frac{x}{8} + \frac{y}{3} + \frac{z}{5}\right)^6};$$

$$5.30. V: \frac{x}{8} + \frac{y}{3} + \frac{z}{5} = 1, \\ x = 0, y = 0, z = 0.$$

Задача 6. Найти площадь фигуры, ограниченной данными линиями.

$$6.1. y = 3/x, y = 4e^x, y = 3, y = 4.$$

$$6.2. x = \sqrt{36 - y^2}, x = 6 - \sqrt{36 - y^2}.$$

$$6.3. x^2 + y^2 = 72, 6y = -x^2 (y \leq 0).$$

$$6.4. x = 8 - y^2, x = -2y.$$

$$6.5. y = \frac{3}{x}, y = 8e^x, y = 3, y = 8.$$

$$6.6. y = \frac{\sqrt{x}}{2}, y = \frac{1}{2x}, x = 16.$$

$$6.7. x = 5 - y^2, x = -4y.$$

$$6.8. x^2 + y^2 = 12, -\sqrt{6}y = x^2 (y \leq 0).$$

$$6.9. y = \sqrt{12 - x^2}, y = 2\sqrt{3} - \sqrt{12 - x^2}, x = 0 (x \geq 0).$$

$$6.10. y = \frac{3}{2}\sqrt{x}, y = \frac{3}{2x}, x = 9.$$

$$6.11. y = \sqrt{24 - x^2}, 2\sqrt{3}y = x^2, x = 0 (x \geq 0).$$

$$6.12. y = \sin x, y = \cos x, x = 0, (x \geq 0).$$

$$6.13. y = 20 - x^2, y = -8x.$$

$$6.14. y = \sqrt{18 - x^2}, y = 3\sqrt{2} - \sqrt{18 - x^2}.$$

$$6.15. y = 32 - x^2, y = -4x.$$

$$6.16. y = 2/x, y = 5e^x, y = 2, y = 5.$$

$$6.17. x^2 + y^2 = 36, 3\sqrt{2}y = x^2 (y \geq 0).$$

$$6.18. y = 3\sqrt{x}, y = 3/x, x = 4.$$

$$6.19. y = 6 - \sqrt{36 - x^2}, y = \sqrt{36 - x^2}, x = 0 (x \geq 0).$$

$$6.20. y = 25/4 - x^2, y = x - 5/2.$$

$$6.21. y = \sqrt{x}, y = 1/x, x = 16.$$

$$6.22. y = 2/x, y = 7e^x, y = 2, y = 7.$$

$$6.23. x = 27 - y^2, x = -6y.$$

$$6.24. x = \sqrt{72 - y^2}, 6x = y^2, y = 0 (y \geq 0).$$

$$6.25. y = \sqrt{6 - x^2}, y = \sqrt{6} - \sqrt{6 - x^2}.$$

$$6.26. y = \frac{3}{2}\sqrt{x}, y = \frac{3}{2x}, x = 4.$$

6.27. $y = \sin x, y = \cos x, x = 0, (x \leq 0)$.

6.28. $y = \frac{1}{x}, y = 6e^x, y = 1, y = 6$.

6.29. $y = 3\sqrt{x}, y = 3/x, x = 9$.

6.30. $y = 11 - x^2, y = -10x$.

6.31. $x^2 + y^2 = 12, x\sqrt{6} = y^2 (x \geq 0)$.

Задача 7. Найти площадь фигуры, ограниченной данными линиями.

$y^2 - 2y + x^2 = 0,$

7.1. $y^2 - 4y + x^2 = 0,$

$y = x/\sqrt{3}, y = \sqrt{3}x.$

$y^2 - 6y + x^2 = 0,$

7.3. $y^2 - 8y + x^2 = 0,$

$y = x/\sqrt{3}, y = \sqrt{3}x.$

$y^2 - 8y + x^2 = 0,$

7.5. $y^2 - 10y + x^2 = 0,$

$y = x/\sqrt{3}, y = \sqrt{3}x.$

$y^2 - 4y + x^2 = 0,$

7.7. $y^2 - 6y + x^2 = 0,$

$y = x, x = 0.$

$y^2 - 6y + x^2 = 0,$

7.9. $y^2 - 10y + x^2 = 0,$

$y = x, x = 0.$

$x^2 - 4x + y^2 = 0,$

7.2. $x^2 - 8x + y^2 = 0,$

$y = 0, y = x/\sqrt{3}.$

$x^2 - 2x + y^2 = 0,$

7.4. $x^2 - 4x + y^2 = 0,$

$y = 0, y = x.$

$x^2 - 4x + y^2 = 0,$

7.6. $x^2 - 8x + y^2 = 0,$

$y = 0, y = x.$

$x^2 - 2x + y^2 = 0,$

7.8. $x^2 - 10x + y^2 = 0,$

$y = 0, y = \sqrt{3}x.$

$x^2 - 2x + y^2 = 0,$

7.10. $x^2 - 4x + y^2 = 0,$

$y = x/\sqrt{3}, y = \sqrt{3}x.$

$$y^2 - 2y + x^2 = 0,$$

7.11. $y^2 - 4y + x^2 = 0,$
 $y = \sqrt{3}x, x = 0.$

$$y^2 - 4y + x^2 = 0,$$

7.13. $y^2 - 6y + x^2 = 0,$
 $y = \sqrt{3}x, x = 0.$

$$y^2 - 2y + x^2 = 0,$$

7.15. $y^2 - 6y + x^2 = 0,$
 $y = x/\sqrt{3}, y = 0.$

$$y^2 - 2y + x^2 = 0,$$

7.17. $y^2 - 10y + x^2 = 0,$
 $y = x/\sqrt{3}, y = \sqrt{3}x.$

$$y^2 - 4y + x^2 = 0,$$

7.19. $y^2 - 10y + x^2 = 0,$
 $y = x/\sqrt{3}, y = \sqrt{3}x.$

$$y^2 - 2y + x^2 = 0,$$

7.21. $y^2 - 4y + x^2 = 0,$
 $y = x, x = 0.$

$$y^2 - 6y + x^2 = 0,$$

7.23. $y^2 - 8y + x^2 = 0,$
 $y = x, x = 0.$

$$y^2 - 4y + x^2 = 0,$$

7.25. $y^2 - 8y + x^2 = 0,$
 $y = x, x = 0.$

$$x^2 - 2x + y^2 = 0,$$

7.12. $x^2 - 6x + y^2 = 0,$
 $y = x/\sqrt{3}, y = \sqrt{3}x.$

$$x^2 - 2x + y^2 = 0,$$

7.14. $x^2 - 8x + y^2 = 0,$
 $y = x/\sqrt{3}, y = \sqrt{3}x.$

$$x^2 - 2x + y^2 = 0,$$

7.16. $x^2 - 4x + y^2 = 0,$
 $y = 0, y = x/\sqrt{3}.$

$$x^2 - 2x + y^2 = 0,$$

7.18. $x^2 - 6x + y^2 = 0,$
 $y = 0, y = x/\sqrt{3}.$

$$x^2 - 2x + y^2 = 0,$$

7.20. $x^2 - 6x + y^2 = 0,$
 $y = 0, y = x.$

$$x^2 - 2x + y^2 = 0,$$

7.22. $x^2 - 4x + y^2 = 0,$
 $y = 0, y = \sqrt{3}x.$

$$x^2 - 4x + y^2 = 0,$$

7.24. $x^2 - 8x + y^2 = 0,$
 $y = 0, y = \sqrt{3}x.$

$$x^2 - 4x + y^2 = 0,$$

7.26. $x^2 - 8x + y^2 = 0,$
 $y = x/\sqrt{3}, y = \sqrt{3}x.$

$$y^2 - 4y + x^2 = 0,$$

$$7.27. \quad y^2 - 8y + x^2 = 0,$$

$$y = \sqrt{3}x, \quad x = 0.$$

$$y^2 - 2y + x^2 = 0,$$

$$7.29. \quad y^2 - 10y + x^2 = 0,$$

$$y = x/\sqrt{3}, \quad x = 0.$$

$$y^2 - 4y + x^2 = 0,$$

$$7.31. \quad y^2 - 8y + x^2 = 0,$$

$$y = x/\sqrt{3}, \quad x = 0.$$

$$x^2 - 4x + y^2 = 0,$$

$$7.28. \quad x^2 - 6x + y^2 = 0,$$

$$y = x/\sqrt{3}, \quad y = \sqrt{3}x.$$

$$x^2 - 6x + y^2 = 0,$$

$$7.30. \quad x^2 - 10x + y^2 = 0,$$

$$y = x/\sqrt{3}, \quad y = \sqrt{3}x.$$

Задача 8. Пластинка D задана ограничивающими ее кривыми, μ - поверхностная плотность. Найти массу пластинки.

$$8.1. \quad D: x=1, y=0, y^2=4x \quad (y \geq 0);$$

$$\mu = 7x^2 + y.$$

$$8.3. \quad D: x=1, y=0, y^2=4x \quad (y \geq 0);$$

$$\mu = 7x^2/2 + 5y.$$

$$8.5. \quad D: x=2, y=0, y^2=2x \quad (y \geq 0);$$

$$\mu = 7x^2/8 + 2y.$$

$$D: x^2 + y^2 = 1, \quad x^2 + y^2 = 4,$$

$$8.2. \quad x=0, y=0 \quad (x \geq 0, y \geq 0);$$

$$\mu = (x+y)/(x^2 + y^2).$$

$$D: x^2 + y^2 = 9, \quad x^2 + y^2 = 16,$$

$$8.4. \quad x=0, y=0 \quad (x \geq 0, y \geq 0);$$

$$\mu = (2x+5y)/(x^2 + y^2).$$

$$D: x^2 + y^2 = 1, \quad x^2 + y^2 = 16,$$

$$8.6. \quad x=0, y=0 \quad (x \geq 0, y \geq 0);$$

$$\mu = (x+y)/(x^2 + y^2).$$

8.7. $D: x=2, y=0, y^2 = x/2 \ (y \geq 0);$
 $\mu = 7x^2/2 + 6y.$

8.9. $D: x=1, y=0, y^2 = 4x \ (y \geq 0);$
 $\mu = x + 3y^2.$

8.11. $D: x=1, y=0, y^2 = x \ (y \geq 0);$
 $\mu = 3x + 6y^2.$

8.13. $D: x=2, y=0, y^2 = x/2 \ (y \geq 0);$
 $\mu = 2x + 3y^2.$

8.15. $D: x = \frac{1}{2}, y=0, y^2 = 8x \ (y \geq 0);$
 $\mu = 7x + 3y^2.$

8.17. $D: x=1, y=0, y^2 = 4x \ (y \geq 0);$
 $\mu = 7x^2 + 2y.$

8.19. $D: x=2, y^2 = 2x, y=0 \ (y \geq 0);$
 $\mu = 7x^2/4 + y/2.$

8.8. $D: x^2 + y^2 = 4, x^2 + y^2 = 25,$
 $x=0, y=0 \ (x \geq 0, y \leq 0);$
 $\mu = (2x - 3y)/(x^2 + y^2).$

8.10. $D: x^2 + y^2 = 1, x^2 + y^2 = 9,$
 $x=0, y=0 \ (x \geq 0, y \leq 0);$
 $\mu = (x - y)/(x^2 + y^2).$

8.12. $D: x^2 + y^2 = 9, x^2 + y^2 = 25,$
 $x=0, y=0 \ (x \leq 0, y \geq 0);$
 $\mu = (2y - x)/(x^2 + y^2).$

8.14. $D: x^2 + y^2 = 4, x^2 + y^2 = 16,$
 $x=0, y=0 \ (x \leq 0, y \geq 0);$
 $\mu = (2y - 3x)/(x^2 + y^2).$

8.16. $D: x^2 + y^2 = 9, x^2 + y^2 = 16,$
 $x=0, y=0 \ (x \leq 0, y \geq 0);$
 $\mu = (2y - 5x)/(x^2 + y^2).$

8.18. $D: x^2 + y^2 = 1, x^2 + y^2 = 16,$
 $x=0, y=0 \ (x \geq 0, y \geq 0);$
 $\mu = (x + 3y)/(x^2 + y^2).$

8.20. $D: x^2 + y^2 = 1, x^2 + y^2 = 4,$
 $x=0, y=0 \ (x \geq 0, y \geq 0);$
 $\mu = (x + 2y)/(x^2 + y^2).$

8.21. $D: x = 2, y = 0, y^2 = 2x \ (y \geq 0);$
 $\mu = 7x^2/4 + y.$

8.22. $D: x^2 + y^2 = 1, x^2 + y^2 = 9,$
 $x = 0, y = 0 \ (x \geq 0, y \leq 0);$
 $\mu = (2x - y)/(x^2 + y^2).$

8.23. $D: x = 2, y = 0, y^2 = x/2 \ (y \geq 0);$
 $\mu = 7x^2/2 + 8y.$

8.24. $D: x^2 + y^2 = 1, x^2 + y^2 = 25,$
 $x = 0, y = 0 \ (x \geq 0, y \leq 0);$
 $\mu = (x - 4y)/(x^2 + y^2).$

8.25. $D: x = 1, y = 0, y^2 = 4x \ (y \geq 0);$
 $\mu = 6x + 3y^2.$

8.26. $D: x^2 + y^2 = 4, x^2 + y^2 = 16,$
 $x = 0, y = 0 \ (x \geq 0, y \leq 0);$
 $\mu = (3x - y)/(x^2 + y^2).$

8.27. $D: x = 2, y = 0, y^2 = x/2 \ (y \geq 0);$
 $\mu = 4x + 6y^2.$

8.28. $D: x^2 + y^2 = 4, x^2 + y^2 = 9,$
 $x = 0, y = 0 \ (x \leq 0, y \geq 0);$
 $\mu = (y - 4x)/(x^2 + y^2).$

8.29. $D: x = \frac{1}{2}, y = 0, y^2 = 2x \ (y \geq 0);$
 $\mu = 4x + 9y^2.$

8.30. $D: x^2 + y^2 = 4, x^2 + y^2 = 9,$
 $x = 0, y = 0 \ (x \leq 0, y \geq 0);$
 $\mu = (y - 2x)/(x^2 + y^2).$

8.31. $D: x = \frac{1}{4}, y = 0, y^2 = 16x \ (y \geq 0);$
 $\mu = 16x + 9y^2/2.$

Задача 9. Пластинка D задана неравенствами, μ - поверхностная плотность. Найти массу пластинки.

9.1. $D: x^2 + y^2/4 \leq 1;$
 $\mu = y^2.$

9.3. $D: x^2/9 + y^2/25 \leq 1;$
 $y \geq 0;$
 $\mu = x^2 y.$

9.5. $D: 1 \leq x^2/9 + y^2/4 \leq 4;$
 $y \geq 0, y \leq x/2;$
 $\mu = 8y/x^3.$

9.7. $D: x^2/4 + y^2 \leq 1;$
 $\mu = 4y^4.$

9.9. $D: 1 \leq x^2/16 + y^2/4 \leq 4;$
 $x \geq 0, y \leq x/2;$
 $\mu = x/y.$

9.11. $D: x^2/4 + y^2 \leq 1;$
 $x \geq 0, y \geq 0;$
 $\mu = 6x^3 y^3.$

9.13. $D: x^2/9 + y^2/4 \leq 1;$
 $\mu = x^2 y^2.$

9.15. $D: x^2/4 + y^2 \leq 1;$
 $x \geq 0, y \geq 0;$
 $\mu = 30x^3 y^7.$

$D: 1 \leq x^2/9 + y^2/4 \leq 2;$

9.2. $y \geq 0, y \leq \frac{2}{3}x;$
 $\mu = y/x.$

9.4. $D: x^2/9 + y^2/25 \leq 1;$
 $y \geq 0;$
 $\mu = 7x^2 y/18.$

9.6. $D: x^2/9 + y^2 \leq 1;$
 $x \geq 0;$
 $\mu = 7xy^6.$

9.8. $D: 1 \leq x^2/4 + y^2/9 \leq 4;$
 $x \geq 0, y \leq 3x/2;$
 $\mu = x/y.$

9.10. $D: x^2/4 + y^2/9 \leq 1;$
 $x \geq 0, y \geq 0;$
 $\mu = x^3 y.$

9.12. $D: 1 \leq x^2/4 + y^2 \leq 25;$
 $x \geq 0, y \leq x/2;$
 $\mu = x/y^3.$

9.14. $D: x^2/16 + y^2 \leq 1;$
 $x \geq 0, y \geq 0;$
 $\mu = 5xy^7.$

9.16. $D: 1 \leq x^2/9 + y^2/4 \leq 3;$
 $y \geq 0, y \leq \frac{2}{3}x;$
 $\mu = y/x.$

$$D: x^2 + y^2/25 \leq 1;$$

9.17. $y \geq 0;$
 $\mu = 7x^4 y.$

$$D: x^2 + y^2/9 \leq 1;$$

9.18. $y \geq 0;$
 $\mu = 35x^4 y^3.$

$$D: x^2/4 + y^2/9 \leq 1;$$

9.19. $\mu = x^2.$

$$D: 1 \leq x^2 + y^2/16 \leq 9;$$

9.20. $y \geq 0, y \leq 4x;$
 $\mu = y/x^3.$

$$D: x^2/9 + y^2 \leq 1;$$

9.21. $x \geq 0;$
 $\mu = 11xy^8.$

$$D: 1 \leq x^2/4 + y^2/16 \leq 5;$$

9.22. $x \geq 0, y \leq 2x;$
 $\mu = x/y.$

$$D: 1 \leq x^2/9 + y^2/4 \leq 5;$$

9.23. $x \geq 0, y \leq 2x/3;$
 $\mu = x/y.$

$$D: x^2/4 + y^2/9 \leq 1;$$

9.24. $x \geq 0, y \geq 0;$
 $\mu = x^5 y.$

$$D: x^2/4 + y^2/25 \leq 1;$$

9.25. $\mu = x^4.$

$$D: x^2 + y^2/4 \leq 1;$$

9.26. $x \geq 0, y \geq 0;$
 $\mu = 15x^5 y^3.$

$$D: 1 \leq x^2/4 + y^2/9 \leq 36;$$

9.27. $x \geq 0, y \geq \frac{3}{2}x;$
 $\mu = 9x/y^3.$

$$D: x^2/100 + y^2 \leq 1;$$

9.28. $x \geq 0, y \geq 0;$
 $\mu = 6xy^9.$

$$D: x^2/16 + y^2 \leq 1;$$

9.29. $x \geq 0, y \geq 0;$
 $\mu = 105x^3 y^9.$

$$D: 1 \leq x^2/9 + y^2/16 \leq 2;$$

9.30. $y \geq 0, y \leq \frac{4}{3}x;$
 $\mu = 27y/x^5.$

$$D: 1 \leq x^2/16 + y^2 \leq 3;$$

9.31. $x \geq 0, y \geq x/4;$
 $\mu = x/y^5.$

Задача 10. Найти объем тела, заданного ограничивающими его поверхностями.

10.1. $y = 16\sqrt{2x}, y = \sqrt{2x},$
 $z = 0, x + z = 2.$

10.2. $y = 5\sqrt{x}, y = 5x/3,$
 $z = 0, z = 5 + 5\sqrt{x}/3.$

10.3. $x^2 + y^2 = 2, y = \sqrt{x}, y = 0,$
 $z = 0, z = 15x.$

10.4. $x + y = 2, y = \sqrt{x},$
 $z = 12y, z = 0.$

10.5. $x = 20\sqrt{2y}, x = 5\sqrt{2y},$
 $z = 0, z + y = 1/2.$

10.6. $x = 5\sqrt{y}/2, x = 5y/6,$
 $z = 0, z = \frac{5}{6}(3 + \sqrt{y}).$

10.7. $x^2 + y^2 = 2, x = \sqrt{y}, x = 0,$
 $z = 0, z = 30y.$

10.8. $x + y = 2, x = \sqrt{y},$
 $z = 12x/5, z = 0.$

10.9. $y = 17\sqrt{2x}, y = 2\sqrt{2x},$
 $z = 0, x + z = 1/2.$

10.10. $y = 5\sqrt{x}/3, y = 5x/9,$
 $z = 0, z = 5(3 + \sqrt{x})/9.$

10.11. $x^2 + y^2 = 8, y = \sqrt{2x}, y = 0,$
 $z = 0, z = 15x/11.$

10.12. $x + y = 4, y = \sqrt{2x},$
 $z = 3y, z = 0.$

10.13. $x = \frac{5}{6}\sqrt{y}, x = \frac{5}{18}y,$
 $z = 0, z = \frac{5}{18}(3 + \sqrt{y}).$

10.14. $x = 19\sqrt{2y}, x = 4\sqrt{2y},$
 $z = 0, z + y = 2.$

10.15. $x^2 + y^2 = 8, x = \sqrt{2y}, x = 0,$
 $z = 30y/11, z = 0.$

10.16. $x + y = 4, x = \sqrt{2y},$
 $z = 3x/5, z = 0.$

10.17. $y = 6\sqrt{3x}, y = \sqrt{3x},$
 $z = 0, x + z = 3.$

10.18. $y = \frac{5}{6}\sqrt{x}, y = \frac{5}{18}x,$
 $z = 0, z = \frac{5}{18}(3 + \sqrt{x}).$

$$10.19. \quad x^2 + y^2 = 18, \quad y = \sqrt{3x}, \quad y = 0, \\ z = 0, \quad z = 5x/11.$$

$$10.20. \quad x + y = 6, \quad y = \sqrt{3x}, \\ z = 4y, \quad z = 0.$$

$$10.21. \quad x = 7\sqrt{3y}, \quad x = 2\sqrt{3y}, \\ z = 0, \quad z + y = 3.$$

$$10.22. \quad x = 5\sqrt{y}/3, \quad x = 5y/9, \\ z = 0, \quad z = 5(3 + \sqrt{y})/9.$$

$$10.23. \quad x^2 + y^2 = 18, \quad x = \sqrt{3y}, \quad x = 0, \\ z = 0, \quad z = 10y/11.$$

$$10.24. \quad x + y = 6, \quad x = \sqrt{3y}, \\ z = 4x/5, \quad z = 0.$$

$$10.25. \quad y = \sqrt{15x}, \quad y = \sqrt{15x}, \\ z = 0, \quad z = \sqrt{15}(1 + \sqrt{x}).$$

$$10.26. \quad x^2 + y^2 = 50, \quad y = \sqrt{5x}, \\ y = 0, \quad z = 0, \quad z = 3x/11.$$

$$10.27. \quad x + y = 8, \quad y = \sqrt{4x}, \\ z = 3y, \quad z = 0.$$

$$10.28. \quad x = 16\sqrt{2y}, \quad x = \sqrt{2y}, \\ z + y = 2, \quad z = 0.$$

$$10.29. \quad x = \sqrt{y}, \quad x = 15y, \\ z = 0, \quad z = 15(1 + \sqrt{y}).$$

$$10.30. \quad x^2 + y^2 = 50, \quad x = \sqrt{5y}, \\ x = 0, \quad z = 0, \quad z = 6y/11.$$

$$10.31. \quad x = 17\sqrt{2y}, \quad x = 2\sqrt{2y}, \\ z = 0, \quad z + y = 1/2.$$

Задача 11. Найти объем тела, заданного ограничивающими его поверхностями.

$$11.1. \quad x^2 + y^2 = 2y, \\ z = 5/4 - x^2, \quad z = 0.$$

$$11.2. \quad x^2 + y^2 = y, \quad x^2 + y^2 = 4y, \\ z = \sqrt{x^2 + y^2}, \quad z = 0.$$

$$11.3. \quad x^2 + y^2 = 8\sqrt{2}x, \\ z = x^2 + y^2 - 64, \\ z = 0 \quad (z \geq 0).$$

$$11.4. \quad x^2 + y^2 + 4x = 0, \\ z = 8 - y^2, \quad z = 0.$$

$$x^2 + y^2 = 6x, \quad x^2 + y^2 = 9x,$$

11.5. $z = \sqrt{x^2 + y^2}, \quad z = 0,$
 $y = 0 \quad (y \leq 0)$

11.7. $x^2 + y^2 = 2y,$
 $z = 9/4 - x^2, \quad z = 0.$

$$x^2 + y^2 + 2\sqrt{2}y = 0,$$

11.9. $z = x^2 + y^2 - 4,$
 $z = 0 \quad (z \geq 0).$

$$x^2 + y^2 = 7x, \quad x^2 + y^2 = 9x,$$

11.11. $z = \sqrt{x^2 + y^2}, \quad z = 0,$
 $y = 0 \quad (y \leq 0)$

11.13. $x^2 + y^2 = 2y,$
 $z = 13/4 - x^2, \quad z = 0.$

$$x^2 + y^2 = 6\sqrt{2}x,$$

11.15. $z = x^2 + y^2 - 36,$
 $z = 0 \quad (z \geq 0).$

11.17. $x^2 + y^2 = 4x,$
 $z = 12 - y^2, \quad z = 0.$

$$x^2 + y^2 = 4\sqrt{2}x,$$

11.19. $z = x^2 + y^2 - 16,$
 $z = 0 \quad (z \geq 0).$

$$x^2 + y^2 = 6\sqrt{2}y,$$

11.6. $z = x^2 + y^2 - 36,$
 $z = 0 \quad (z \geq 0).$

11.8. $x^2 + y^2 = 2y, \quad x^2 + y^2 = 5y,$
 $z = \sqrt{x^2 + y^2}, \quad z = 0.$

11.10. $x^2 + y^2 = 4x,$
 $z = 10 - y^2, \quad z = 0.$

$$x^2 + y^2 = 8\sqrt{2}y,$$

11.12. $z = x^2 + y^2 - 64,$
 $z = 0 \quad (z \geq 0).$

11.14. $x^2 + y^2 = 3y, \quad x^2 + y^2 = 6y,$
 $z = \sqrt{x^2 + y^2}, \quad z = 0.$

$$x^2 + y^2 = 2\sqrt{2}y,$$

11.16. $z = x^2 + y^2 - 4,$
 $z = 0 \quad (z \geq 0).$

$$x^2 + y^2 = 8x, \quad x^2 + y^2 = 11x,$$

11.18. $z = \sqrt{x^2 + y^2}, \quad z = 0,$
 $y = 0 \quad (y \leq 0)$

11.20. $x^2 + y^2 = 4y,$
 $z = 4 - x^2, \quad z = 0.$

$$11.21. \quad \begin{aligned} x^2 + y^2 &= 4y, \quad x^2 + y^2 = 7y, \\ z &= \sqrt{x^2 + y^2}, \quad z = 0. \end{aligned}$$

$$11.23. \quad \begin{aligned} x^2 + y^2 + 2x &= 0, \\ z &= 17/4 - y^2, \quad z = 0. \end{aligned}$$

$$11.25. \quad \begin{aligned} x^2 + y^2 + 2\sqrt{2}x &= 0, \\ z &= x^2 + y^2 - 4, \\ z &= 0 \quad (z \geq 0). \end{aligned}$$

$$11.27. \quad \begin{aligned} x^2 + y^2 &= 10x, \quad x^2 + y^2 = 13x, \\ z &= \sqrt{x^2 + y^2}, \quad z = 0, \\ y &= 0 \quad (y \geq 0) \end{aligned}$$

$$11.29. \quad \begin{aligned} x^2 + y^2 &= 2x, \\ z &= 21/4 - y^2, \quad z = 0. \end{aligned}$$

$$11.31. \quad \begin{aligned} x^2 + y^2 + 2x &= 0, \\ z &= 25/4 - y^2, \quad z = 0. \end{aligned}$$

$$11.22. \quad \begin{aligned} x^2 + y^2 &= 4\sqrt{2}y, \\ z &= x^2 + y^2 - 16, \\ z &= 0 \quad (z \geq 0). \end{aligned}$$

$$11.24. \quad \begin{aligned} x^2 + y^2 &= 9x, \quad x^2 + y^2 = 12x, \\ z &= \sqrt{x^2 + y^2}, \quad z = 0, \\ y &= 0 \quad (y \geq 0) \end{aligned}$$

$$11.26. \quad \begin{aligned} x^2 + y^2 &= 4y, \\ z &= 6 - x^2, \quad z = 0. \end{aligned}$$

$$11.28. \quad \begin{aligned} x^2 + y^2 &= 2\sqrt{2}x, \\ z &= x^2 + y^2 - 4, \\ z &= 0 \quad (z \geq 0). \end{aligned}$$

$$11.30. \quad \begin{aligned} x^2 + y^2 &= 5y, \quad x^2 + y^2 = 8y, \\ z &= \sqrt{x^2 + y^2}, \quad z = 0. \end{aligned}$$

Задача 12. Найти объем тела, заданного ограничивающими его поверхностями.

$$12.1. \quad \begin{aligned} y &= 5x^2 + 2, \quad y = 7, \\ z &= 3y^2 - 7x^2 - 2, \\ z &= 3y^2 - 7x^2 - 5. \end{aligned}$$

$$12.2. \quad \begin{aligned} y &= 5x^2 - 2, \quad y = -4x^2 + 7, \\ z &= 4 + 9x^2 + 5y^2, \\ z &= -1 + 9x^2 + 5y^2. \end{aligned}$$

$$x = -5y^2 + 2, \quad x = -3,$$

$$12.3. \quad z = 3x^2 + y^2 + 1,$$

$$z = 3x^2 + y^2 - 5.$$

$$y = -6x^2 + 8, \quad y = 2,$$

$$12.5. \quad z = x - x^2 - y^2 - 1,$$

$$z = x - x^2 - y^2 - 5.$$

$$x = 5y^2 - 9, \quad x = -4,$$

$$12.7. \quad z = x^2 + 4x - y^2 - 4,$$

$$z = x^2 + 4x - y^2 + 2.$$

$$x = 5y^2 - 1, \quad x = -3y^2 + 1,$$

$$12.9. \quad z = 2 - \sqrt{x^2 + 6y^2},$$

$$z = -1 - \sqrt{x^2 + 6y^2}.$$

$$y = -5x^2 + 3, \quad y = -2,$$

$$12.11. \quad z = 2x^2 - 3y - 6y^2 - 1,$$

$$z = 2x^2 - 3y - 6y^2 + 2.$$

$$x = 3y^2 - 5, \quad x = -2,$$

$$12.13. \quad z = 2 - \sqrt{x^2 + 16y^2},$$

$$z = 8 - \sqrt{x^2 + 16y^2}.$$

$$y = 2x^2 - 1, \quad y = 1,$$

$$12.15. \quad z = x^2 - 5y^2 - 3,$$

$$z = x^2 - 5y^2 - 6.$$

$$x = 2y^2 - 3, \quad x = -7y^2 + 6,$$

$$12.4. \quad z = 1 + \sqrt{x^2 + 16y^2},$$

$$z = -3 + \sqrt{x^2 + 16y^2}.$$

$$y = 5x^2 - 1, \quad y = -3x^2 + 1,$$

$$12.6. \quad z = -2 + \sqrt{3x^2 + y^2},$$

$$z = -5 + \sqrt{3x^2 + y^2}.$$

$$y = 6x^2 - 1, \quad y = 5,$$

$$12.8. \quad z = 2x^2 + x - y^2,$$

$$z = 2x^2 + x - y^2 + 4.$$

$$x = -3y^2 + 7, \quad x = 4,$$

$$12.10. \quad z = 2 + \sqrt{6x^2 + y^2},$$

$$z = 3 + \sqrt{6x^2 + y^2}.$$

$$y = x^2 - 5, \quad y = -x^2 + 3,$$

$$12.12. \quad z = 4 + \sqrt{5x^2 + 8y^2},$$

$$z = 1 + \sqrt{5x^2 + 8y^2}.$$

$$x = y^2 - 2, \quad x = -4y^2 + 3,$$

$$12.14. \quad z = \sqrt{16 - x^2 - y^2} + 2,$$

$$z = \sqrt{16 - x^2 - y^2} - 1.$$

$$y = x^2 - 2, \quad y = -4x^2 + 3,$$

$$12.16. \quad z = 2 + \sqrt{x^2 + y^2},$$

$$z = -1 + \sqrt{x^2 + y^2}.$$

$$x = -4y^2 + 1, \quad x = -3,$$

$$12.17. \quad z = x^2 - 7y^2 - 1,$$

$$z = x^2 - 7y^2 + 2.$$

$$y = 1 - 2x^2, \quad y = -1,$$

$$12.19. \quad z = x^2 + 2y + y^2 - 2,$$

$$z = x^2 + 2y + y^2 + 1.$$

$$x = 2y^2 + 3, \quad x = 5,$$

$$12.21. \quad z = 1 + \sqrt{9x^2 + 4y^2},$$

$$z = 4 + \sqrt{9x^2 + 4y^2}.$$

$$x = 5y^2 - 2, \quad x = -4y^2 + 7,$$

$$12.23. \quad z = 4 - \sqrt{2x^2 + 3y^2},$$

$$z = -1 - \sqrt{2x^2 + 3y^2}.$$

$$y = -3x^2 + 5, \quad y = 2,$$

$$12.25. \quad z = 3 + \sqrt{5x^2 + y^2},$$

$$z = -1 + \sqrt{5x^2 + y^2}.$$

$$x = 4y^2 + 2, \quad x = 6,$$

$$12.27. \quad z = x^2 + 4y^2 + y + 1,$$

$$z = x^2 + 4y^2 + y + 4.$$

$$y = 2x^2 - 5, \quad y = -3,$$

$$12.29. \quad z = 2 + \sqrt{x^2 + 4y^2},$$

$$z = -1 + \sqrt{x^2 + 4y^2}.$$

$$x = 7y^2 - 6, \quad x = -2y^2 + 3,$$

$$12.18. \quad z = 3 - 12y^2 + 5x^2,$$

$$z = -2 - 12y^2 + 5x^2.$$

$$y = x^2 - 7, \quad y = -8x^2 + 2,$$

$$12.20. \quad z = 3 - 12y^2 + 5x^2,$$

$$z = -2 - 12y^2 + 5x^2.$$

$$y = 3x^2 + 4, \quad y = 7,$$

$$12.22. \quad z = 5 - \sqrt{2x^2 + 3y^2},$$

$$z = 1 - \sqrt{2x^2 + 3y^2}.$$

$$x = -2y^2 + 5, \quad x = 3,$$

$$12.24. \quad z = 5 - \sqrt{x^2 + 25y^2},$$

$$z = 2 - \sqrt{x^2 + 25y^2}.$$

$$y = 3x^2 - 5, \quad y = -6x^2 + 4,$$

$$12.26. \quad z = 2 + 10x^2 - y^2,$$

$$z = -2 + 10y^2 - y^2.$$

$$x = 3y^2 - 2, \quad x = -4y^2 + 5,$$

$$12.28. \quad z = 4 - 7x^2 - 9y^2,$$

$$z = 1 - 7x^2 - 9y^2.$$

$$y = 2x^2 - 3, \quad y = -7x^2 + 6,$$

$$12.30. \quad z = 1 - 5x^2 - 6y^2,$$

$$z = -3 - 5x^2 - 6y^2.$$

$$y = -2x^2 + 7, \quad y = 5,$$

$$12.31. \quad z = 1 - 2x^2 + 3y^2,$$

$$z = 4 - 2x^2 + 3y^2.$$

Задача 13. Найти объем тела, заданного ограничивающими его поверхностями

$$13.1. \quad z = \sqrt{9 - x^2 - y^2},$$

$$9z/2 = x^2 + y^2.$$

$$13.2. \quad z = 15\sqrt{x^2 + y^2}/2,$$

$$z = 17/2 - x^2 - y^2.$$

$$13.3. \quad z = \sqrt{4 - x^2 - y^2},$$

$$z = \sqrt{(x^2 + y^2)/255}.$$

$$z = \sqrt{64 - x^2 - y^2}, \quad z = 1,$$

$$13.4. \quad x^2 + y^2 = 60$$

(внутри цилиндра).

$$13.5. \quad z = \sqrt{\frac{16}{9} - x^2 - y^2},$$

$$2z = x^2 + y^2.$$

$$13.6. \quad z = 3\sqrt{x^2 + y^2},$$

$$z = 10 - x^2 - y^2.$$

$$13.7. \quad z = \sqrt{25 - x^2 - y^2},$$

$$z = \sqrt{(x^2 + y^2)/99}.$$

$$z = \sqrt{100 - x^2 - y^2}, \quad z = 6,$$

$$13.8. \quad x^2 + y^2 = 51$$

(внутри цилиндра).

$$13.9. \quad z = 21\sqrt{x^2 + y^2}/2,$$

$$z = 23/2 - x^2 - y^2.$$

$$13.10. \quad z = \sqrt{16 - x^2 - y^2},$$

$$6z = x^2 + y^2.$$

$$13.11. \quad z = \sqrt{9 - x^2 - y^2},$$

$$z = \sqrt{(x^2 + y^2)/80}.$$

$$z = \sqrt{81 - x^2 - y^2}, \quad z = 5,$$

$$13.12. \quad x^2 + y^2 = 45$$

(внутри цилиндра).

$$13.13. \quad z = \sqrt{1 - x^2 - y^2},$$

$$3z/2 = x^2 + y^2.$$

$$13.14. \quad z = 6\sqrt{x^2 + y^2},$$

$$z = 16 - x^2 - y^2.$$

$$13.15. \quad z = \sqrt{36 - x^2 - y^2}, \\ z = \sqrt{(x^2 + y^2)}/63.$$

$$13.17. \quad z = \sqrt{144 - x^2 - y^2}, \\ 18z = x^2 + y^2.$$

$$13.19. \quad z = \sqrt{9 - x^2 - y^2}, \\ z = \sqrt{(x^2 + y^2)}/35.$$

$$13.21. \quad z = \sqrt{36 - x^2 - y^2}, \\ 9z = x^2 + y^2.$$

$$13.23. \quad z = \sqrt{16 - x^2 - y^2}, \\ z = \sqrt{(x^2 + y^2)}/15.$$

$$13.25. \quad z = \sqrt{4/9 - x^2 - y^2}, \\ z = x^2 + y^2.$$

$$13.27. \quad z = \sqrt{9 - x^2 - y^2}, \\ z = \sqrt{(x^2 + y^2)}/8.$$

$$13.29. \quad z = \sqrt{64 - x^2 - y^2}, \\ 12z = x^2 + y^2.$$

$$13.31. \quad z = \sqrt{36 - x^2 - y^2}, \\ z = \sqrt{(x^2 + y^2)}/3.$$

$$z = \sqrt{64 - x^2 - y^2}, \quad z = 4, \\ 13.16. \quad x^2 + y^2 = 39 \\ \text{(внутри цилиндра).}$$

$$13.18. \quad z = 3\sqrt{x^2 + y^2}/2, \\ z = 5/2 - x^2 - y^2.$$

$$z = \sqrt{49 - x^2 - y^2}, \quad z = 3, \\ 13.20. \quad x^2 + y^2 = 33 \\ \text{(внутри цилиндра).}$$

$$13.22. \quad z = 9\sqrt{x^2 + y^2}, \\ z = 22 - x^2 - y^2.$$

$$z = \sqrt{36 - x^2 - y^2}, \quad z = 2, \\ 13.24. \quad x^2 + y^2 = 27 \\ \text{(внутри цилиндра).}$$

$$13.26. \quad z = 12\sqrt{x^2 + y^2}, \\ z = 28 - x^2 - y^2.$$

$$z = \sqrt{25 - x^2 - y^2}, \quad z = 1, \\ 13.28. \quad x^2 + y^2 = 21 \\ \text{(внутри цилиндра).}$$

$$13.30. \quad z = 9\sqrt{x^2 + y^2}/2, \\ z = 11/2 - x^2 - y^2.$$

Задача 14. Найти объем тела, заданного ограничивающими его поверхностями.

14.1. $z = 2 - 12(x^2 + y^2),$
 $z = 24x + 2.$

14.3. $z = 8(x^2 + y^2) + 3,$
 $z = 16x + 3.$

14.5. $z = 4 - 14(x^2 + y^2),$
 $z = 4 - 28x.$

14.7. $z = 32(x^2 + y^2) + 3,$
 $z = 3 - 64x.$

14.9. $z = 2 - 4(x^2 + y^2),$
 $z = 8x + 2.$

14.11. $z = 24(x^2 + y^2) + 1,$
 $z = 48x + 1.$

14.13. $z = -16(x^2 + y^2) - 1,$
 $z = -32x - 1.$

14.15. $z = 26(x^2 + y^2) - 2,$
 $z = -52x - 2.$

14.17. $z = -2(x^2 + y^2) - 1,$
 $z = 4y - 1.$

14.2. $z = 10[(x-1)^2 + y^2] + 1,$
 $z = 21 - 20x.$

14.4. $z = 2 - 20[(x+1)^2 + y^2],$
 $z = -40 - 38x.$

14.6. $z = 28[(x+1)^2 + y^2] + 3,$
 $z = 56x + 59.$

14.8. $z = 4 + 6[(x-1)^2 + y^2],$
 $z = -12x - 8.$

14.10. $z = 22[(x-1)^2 + y^2] + 3,$
 $z = 47 - 44x.$

14.12. $z = 2 - 18[(x+1)^2 + y^2],$
 $z = -36x - 34.$

14.14. $z = 30[(x+1)^2 + y^2] + 1,$
 $z = 60x + 61.$

14.16. $z = -2[(x-1)^2 + y^2] - 1,$
 $z = 4x - 5.$

14.18. $z = 26[(x-1)^2 + y^2] - 2,$
 $z = 50 - 52x.$

$$14.19. \begin{aligned} z &= 30(x^2 + y^2) + 1, \\ z &= 60y + 1. \end{aligned}$$

$$14.21. \begin{aligned} z &= 2 - 18(x^2 + y^2), \\ z &= 2 - 36y. \end{aligned}$$

$$14.23. \begin{aligned} z &= 22(x^2 + y^2) + 3, \\ z &= 3 - 44y. \end{aligned}$$

$$14.25. \begin{aligned} z &= 4 - 6(x^2 + y^2), \\ z &= 12y + 4. \end{aligned}$$

$$14.27. \begin{aligned} z &= 28(x^2 + y^2) + 3, \\ z &= 56y + 3. \end{aligned}$$

$$14.29. \begin{aligned} z &= 2 - 20(x^2 + y^2), \\ z &= 2 - 40y. \end{aligned}$$

$$14.31. \begin{aligned} z &= 10(x^2 + y^2) + 1, \\ z &= 1 - 20y. \end{aligned}$$

$$14.20. \begin{aligned} z &= -16[(x+1)^2 + y^2] - 1, \\ z &= -32x - 33. \end{aligned}$$

$$14.22. \begin{aligned} z &= 24[(x+1)^2 + y^2] + 1, \\ z &= 48x + 49. \end{aligned}$$

$$14.24. \begin{aligned} z &= 2 - 4[(x-1)^2 + y^2], \\ z &= 8x - 6. \end{aligned}$$

$$14.26. \begin{aligned} z &= 32[(x-1)^2 + y^2] + 3, \\ z &= 67 - 64x. \end{aligned}$$

$$14.28. \begin{aligned} z &= 4 - 14[(x+1)^2 + y^2], \\ z &= -28x - 24. \end{aligned}$$

$$14.30. \begin{aligned} z &= 8[(x+1)^2 + y^2] + 3, \\ z &= 16x + 19. \end{aligned}$$

Задача 15. Найти объем тела, заданного неравенствами.

$$15.1. \begin{aligned} 1 &\leq x^2 + y^2 + z^2 \leq 49, \\ -\sqrt{\frac{x^2 + y^2}{35}} &\leq z \leq \sqrt{\frac{x^2 + y^2}{3}}, \\ x &\leq y \leq 0. \end{aligned}$$

$$15.3. \begin{aligned} 4 &\leq x^2 + y^2 + z^2 \leq 64, \\ z &\leq \sqrt{\frac{x^2 + y^2}{3}}, \quad -\frac{x}{\sqrt{3}} \leq y \leq 0. \end{aligned}$$

$$15.2. \begin{aligned} 1 &\leq x^2 + y^2 + z^2 \leq 64, \\ \sqrt{\frac{x^2 + y^2}{15}} &\leq z \leq \sqrt{\frac{x^2 + y^2}{3}}, \\ -\sqrt{3}x &\leq y \leq 0. \end{aligned}$$

$$15.4. \begin{aligned} 4 &\leq x^2 + y^2 + z^2 \leq 36, \\ z &\geq -\sqrt{\frac{x^2 + y^2}{63}}, \quad 0 \leq y \leq -\frac{x}{\sqrt{3}}. \end{aligned}$$

$$1 \leq x^2 + y^2 + z^2 \leq 36,$$

$$15.5. \quad z \geq \sqrt{\frac{x^2 + y^2}{99}}, \quad -\sqrt{3}x \leq y \leq \sqrt{3}x.$$

$$1 \leq x^2 + y^2 + z^2 \leq 49,$$

$$15.7. \quad 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}},$$

$$y \leq -\frac{x}{\sqrt{3}}, \quad y \leq -\sqrt{3}x.$$

$$4 \leq x^2 + y^2 + z^2 \leq 64,$$

$$15.9. \quad -\sqrt{\frac{x^2 + y^2}{35}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}},$$

$$x \leq y \leq 0.$$

$$16 \leq x^2 + y^2 + z^2 \leq 100,$$

$$15.11. \quad z \leq \sqrt{\frac{x^2 + y^2}{3}}, \quad -\sqrt{3}x \leq y \leq -\frac{x}{\sqrt{3}}.$$

$$4 \leq x^2 + y^2 + z^2 \leq 49,$$

$$15.13. \quad z \geq \sqrt{\frac{x^2 + y^2}{99}}, \quad y \leq 0, \quad y \leq \sqrt{3}x.$$

$$4 \leq x^2 + y^2 + z^2 \leq 64,$$

$$15.15. \quad 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}},$$

$$y \leq \sqrt{3}x, \quad y \leq \frac{x}{\sqrt{3}}.$$

$$25 \leq x^2 + y^2 + z^2 \leq 100,$$

$$15.6. \quad z \leq -\sqrt{\frac{x^2 + y^2}{99}}, \quad \sqrt{3}x \leq y \leq -\sqrt{3}x.$$

$$25 \leq x^2 + y^2 + z^2 \leq 49,$$

$$15.8. \quad -\sqrt{\frac{x^2 + y^2}{24}} \leq z \leq 0,$$

$$y \geq -\frac{x}{\sqrt{3}}, \quad y \geq -\sqrt{3}x.$$

$$16 \leq x^2 + y^2 + z^2 \leq 100,$$

$$15.10. \quad \sqrt{\frac{x^2 + y^2}{15}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}},$$

$$\sqrt{3}x \leq y \leq 0.$$

$$16 \leq x^2 + y^2 + z^2 \leq 64,$$

$$15.12. \quad z \geq -\sqrt{\frac{x^2 + y^2}{63}},$$

$$-\frac{x}{\sqrt{3}} \leq y \leq -\sqrt{3}x.$$

$$36 \leq x^2 + y^2 + z^2 \leq 121,$$

$$15.14. \quad z \geq -\sqrt{\frac{x^2 + y^2}{99}}, \quad y \geq 0, \quad y \geq \sqrt{3}x.$$

$$36 \leq x^2 + y^2 + z^2 \leq 144,$$

$$15.16. \quad -\sqrt{\frac{x^2 + y^2}{24}} \leq z \leq 0,$$

$$y \geq \sqrt{3}x, \quad y \geq \frac{x}{\sqrt{3}}.$$

$$9 \leq x^2 + y^2 + z^2 \leq 81,$$

$$15.17. -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{35}},$$

$$0 \leq y \leq -x.$$

$$36 \leq x^2 + y^2 + z^2 \leq 144,$$

$$15.18. -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{35}},$$

$$0 \leq y \leq -\sqrt{3}x.$$

$$36 \leq x^2 + y^2 + z^2 \leq 144,$$

$$15.19. z \leq \sqrt{\frac{x^2 + y^2}{3}}, \quad \sqrt{3}x \leq y \leq \frac{x}{\sqrt{3}}.$$

$$36 \leq x^2 + y^2 + z^2 \leq 100,$$

$$15.20. z \geq -\sqrt{\frac{x^2 + y^2}{63}},$$

$$\frac{x}{\sqrt{3}} \leq y \leq \sqrt{3}x.$$

$$9 \leq x^2 + y^2 + z^2 \leq 64,$$

$$15.21. z \geq \sqrt{\frac{x^2 + y^2}{99}},$$

$$y \leq \frac{x}{\sqrt{3}}, \quad y \leq -\frac{x}{\sqrt{3}}.$$

$$49 \leq x^2 + y^2 + z^2 \leq 144,$$

$$15.22. z \leq -\sqrt{\frac{x^2 + y^2}{99}},$$

$$y \geq \frac{x}{\sqrt{3}}, \quad y \geq -\frac{x}{\sqrt{3}}.$$

$$9 \leq x^2 + y^2 + z^2 \leq 81,$$

$$15.23. 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}},$$

$$y \leq 0, \quad y \leq \frac{x}{\sqrt{3}}.$$

$$49 \leq x^2 + y^2 + z^2 \leq 81,$$

$$15.24. -\sqrt{\frac{x^2 + y^2}{24}} \leq z \leq 0,$$

$$y \geq 0, \quad y \geq \frac{x}{\sqrt{3}}.$$

$$16 \leq x^2 + y^2 + z^2 \leq 100,$$

$$15.25. -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{35}},$$

$$0 \leq y \leq x.$$

$$64 \leq x^2 + y^2 + z^2 \leq 196,$$

$$15.26. -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq -\sqrt{\frac{x^2 + y^2}{15}},$$

$$0 \leq y \leq \sqrt{3}x.$$

$$64 \leq x^2 + y^2 + z^2 \leq 196,$$

$$15.27. z \leq \sqrt{\frac{x^2 + y^2}{3}}, \quad \frac{x}{\sqrt{3}} \leq y \leq 0.$$

$$64 \leq x^2 + y^2 + z^2 \leq 144,$$

$$15.28. z \geq -\sqrt{\frac{x^2 + y^2}{63}}, \quad 0 \leq y \leq \frac{x}{\sqrt{3}}.$$

$$16 \leq x^2 + y^2 + z^2 \leq 81,$$

$$15.29. z \geq \sqrt{\frac{x^2 + y^2}{99}},$$

$$y \leq 0, \quad y \leq -\sqrt{3}x.$$

$$64 \leq x^2 + y^2 + z^2 \leq 169,$$

$$15.30. z \leq -\sqrt{\frac{x^2 + y^2}{99}},$$

$$y \geq 0, \quad y \geq -\sqrt{3}x.$$

$$16 \leq x^2 + y^2 + z^2 \leq 100,$$

$$15.31. 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}},$$

$$y \leq 0, \quad y \leq \frac{x}{\sqrt{3}}.$$

Задача 16. Тело V задано ограничивающими его поверхностями, μ - плотность.

Найти массу тела.

$$64(x^2 + y^2) = z^2, \quad x^2 + y^2 = 4,$$

$$16.1. y = 0, \quad z = 0 \quad (y \geq 0, \quad z \geq 0),$$

$$\mu = 5(x^2 + y^2)/4.$$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 1,$$

$$16.2. (x^2 + y^2 \leq 1), \quad x = 0 \quad (x \geq 0);$$

$$\mu = 4|z|.$$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 2z,$$

$$16.3. x = 0, \quad y = 0, \quad z = 0 \quad (x \geq 0, \quad y \geq 0);$$

$$\mu = 10x.$$

$$x^2 + y^2 = \frac{16}{49}z^2, \quad x^2 + y^2 = \frac{4}{7}z,$$

16.4. $x = 0, y = 0, (x \geq 0, y \geq 0);$

$$\mu = 80yz.$$

$$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = 4z^2,$$

16.5. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$

$$\mu = 20z.$$

$$36(x^2 + y^2) = z^2, \quad x^2 + y^2 = 1,$$

16.6. $x = 0, z = 0 (x \geq 0, z \geq 0),$

$$\mu = \frac{5}{6}(x^2 + y^2).$$

$$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 4,$$

16.7. $(x^2 + y^2 \leq 4);$

$$\mu = 2|z|.$$

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 8z,$$

16.8. $x = 0, y = 0, z = 0 (x \geq 0, y \geq 0);$

$$\mu = 5x.$$

$$x^2 + y^2 = \frac{4}{25}z^2, \quad x^2 + y^2 = \frac{2}{5}z,$$

16.9. $x = 0, y = 0, (x \geq 0, y \geq 0);$

$$\mu = 28xz.$$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = z^2,$$

16.10. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$

$$\mu = 6z.$$

$$25(x^2 + y^2) = z^2, \quad x^2 + y^2 = 4,$$

16.11. $x = 0, \quad y = 0, \quad z = 0$
 $(x \geq 0, \quad y \geq 0, \quad z \geq 0),$
 $\mu = 2(x^2 + y^2).$

$$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$$

16.12. $(x^2 + y^2 \leq 4), \quad y = 0 \quad (y \geq 0);$
 $\mu = |z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 6z,$$

16.13. $x = 0, \quad y = 0, \quad z = 0 \quad (x \geq 0, \quad y \geq 0);$
 $\mu = 90y.$

$$x^2 + y^2 = \frac{1}{25}z^2, \quad x^2 + y^2 = \frac{1}{5}z,$$

16.14. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0);$
 $\mu = 14yz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 9z^2,$$

16.15. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0);$
 $\mu = 10z.$

$$9(x^2 + y^2) = z^2, \quad x^2 + y^2 = 4,$$

16.16. $x = 0, \quad y = 0, \quad z = 0$
 $(x \geq 0, \quad y \geq 0, \quad z \geq 0),$
 $\mu = 5(x^2 + y^2)/3.$

$$x^2 + y^2 + z^2 = 4,$$

16.17. $x^2 + y^2 = 1, \quad (x^2 + y^2 \leq 1);$
 $\mu = |z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = z,$$

$$x = 0, \quad y = 0, \quad z = 0,$$

16.18. $(x \geq 0, \quad y \geq 0);$
 $\mu = 10y.$

$$x^2 + y^2 = \frac{1}{49}z^2, \quad x^2 + y^2 = \frac{1}{7}z,$$

16.19. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0);$
 $\mu = 10xz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 4z^2,$$

16.20. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0);$
 $\mu = 10z.$

$$16(x^2 + y^2) = z^2, \quad x^2 + y^2 = 1,$$

16.21. $x = 0, \quad y = 0, \quad z = 0 \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0),$
 $\mu = 5(x^2 + y^2).$

$$x^2 + y^2 + z^2 = 16,$$

16.22. $x^2 + y^2 = 4 \quad (x^2 + y^2 \leq 4);$
 $\mu = |z|.$

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 4z,$$

16.23. $x = 0, \quad y = 0, \quad z = 0 \quad (x \geq 0, \quad y \geq 0);$
 $\mu = 3y.$

$$x^2 + y^2 = z^2, \quad x^2 + y^2 = z,$$

16.24. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0);$
 $\mu = 35yz.$

$$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = z^2,$$

16.25. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$

$$\mu = 32z.$$

$$x^2 + y^2 = z^2, \quad x^2 + y^2 = 4,$$

$$x = 0, \quad y = 0, \quad z = 0$$

16.26. $(x \geq 0, y \geq 0, z \geq 0),$

$$\mu = 5(x^2 + y^2)/2.$$

$$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$$

16.27. $(x^2 + y^2 \leq 4), \quad z = 0 \quad (z \geq 0);$

$$\mu = 2z.$$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 3z,$$

$$x = 0, \quad y = 0, \quad z = 0$$

16.28. $(x \geq 0, y \geq 0);$

$$\mu = 15x.$$

$$x^2 + y^2 = \frac{4}{49}z^2, \quad x^2 + y^2 = \frac{2}{7}z,$$

16.29. $x = 0, y = 0, (x \geq 0, y \geq 0);$

$$\mu = 20xz.$$

$$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 9z^2,$$

16.30. $x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0);$

$$\mu = 5z.$$

$$4(x^2 + y^2) = z^2, \quad x^2 + y^2 = 1,$$

16.31. $y = 0, \quad z = 0 \quad (y \geq 0, z \geq 0),$

$$\mu = 10(x^2 + y^2).$$

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